Enformation Theory — Part III:
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From *Models* to *Control*⋆

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Abstract. It turns out that models in nature are motivated because they make it possible to implement best possible *model-based controls*. The generalized control scheme, or the *monad model*, can be seen as a basis for *all living systems*. Indeed, the key observation is that *everything is driven by entropy pursuit.*

1 Introduction

In Part II, it was observed that systems can be seen as *models*, constructing *filters* that eliminate noise when transferring data from high dimensions to the concentrated low dimension. They shuffle the data projecting it into the *principal subspace* that optimally captures the available enformation. The steady-state $\bar{x}$ of the system is captured in the formula

$$\bar{x} = Q\,\mathcal{E}\left\{\bar{x}\bar{u}^{\mathrm{T}}\right\}\bar{u}, \tag{1}$$

where

$$\bar{u} = u - \mathcal{E}\left\{\bar{x}\bar{u}^{\mathrm{T}}\right\}^{\mathrm{T}} Q\,\bar{x}. \tag{2}$$

Here, the *observed* environment $\bar{u}$ is found when the "exploitation" is subtracted from the *original* environment u; matrix Q contains the *coupling parameters* and $\mathcal{E}$ represents the *emergence operator* (see also [1]).

But why do systems construct models? — It turns out that models are needed to implement good controls. And controls are needed to exhaust enformation from the environment, driving it towards *heat death*, or *entropy maximum*. The control loops can be seen as distributed *entropy pumps*; such dynamic structures can be seen as *elements in all living systems*.

First, in paper I, the focus was on *individuals*, and in paper II it was on *systems*. Now, the emphasis will be on the *environment*: how the systems change their surroundings.

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2 Systems as controllers

So, it seems that the population of neocybernetic computing elements implements a special kind of representation of the properties of the incoming data — but why does it do that? It turns out that the constructed *model of the environment* makes it possible to implement *model-based control* to *maximally exhaust information from the environment*, and, as seen from the local point of view, this makes it possible to *maximally exploit the available resources*.

To understand the above claim, some theoretical concepts first need to be discussed.

Even though everything in neural populations is based on elementary operations, the systemic properties can best be understood in terms of multivariate linear theory and as *mappings between spaces*. Interactions between a system and its environment are mappings between the space of inputs and the space of activities of reacting units. When the dynamic equilibrium is found not only on the signal level but also on the statistical level, the relation between the inputs and the linear units is captured by the *explicit mapping*

$$\phi^{\mathrm{T}} = Q\,\mathcal{E}\left\{\bar{x}\bar{u}^{\mathrm{T}}\right\} = \left(Q^{-1} + \mathcal{E}\left\{\bar{x}\bar{x}^{\mathrm{T}}\right\}\right)^{-1}\mathcal{E}\left\{\bar{x}u^{\mathrm{T}}\right\}, \tag{3}$$

according to formula (1); the latter form is found using simple algebra when the definition of $\bar{u}$ is written out inside the emergence operator in (1). This means that the feedforward mapping can be expressed as $\bar{x} = \phi^{\mathrm{T}}\bar{u}$ and the feedback as $\Delta\bar{u} = \phi\bar{x}$. Further, when the effective mapping from the original, undisturbed u to the system state $\bar{x}$ is solved, so that $\bar{x} = \varphi^{\mathrm{T}}u$, one has the following formulation for this *implicit mapping*, according to the derivations in Part II,

$$\varphi^{\mathrm{T}} = \left(Q^{-1} + \mathcal{E}\left\{\bar{x}\bar{x}^{\mathrm{T}}\right\}\right)^{-1}\mathcal{E}\left\{\bar{x}\bar{u}^{\mathrm{T}}\right\}. \tag{4}$$

Using these notations, one can find new formulations; for example, one can express the eigenvector matrix as an "orthogonalization" of the mappings:

$$\theta^{\mathrm{T}} = \left(\phi^{\mathrm{T}}\phi\right)^{-1/2}\phi^{\mathrm{T}} = \left(\varphi^{\mathrm{T}}\varphi\right)^{-1/2}\varphi^{\mathrm{T}}. \tag{5}$$

However, to truly understand what takes place in the cybernetic loop of units, one needs to take a wider perspective.

Assume that there is some data $\xi(k)$ of dimension n, and there is some other related data $\zeta(k)$ of higher dimension m, with $1 \leq k \leq K$. One would like to find the best possible (approximate) mapping from the space of ξ to the space of ζ so that the average of the squared *reconstruction error*, or $\|\zeta(k) - \hat{\zeta}(k)\|_2^2$, would be minimized (note that now one would like to find the optimal mapping from the lower to the higher dimension, whereas in principal component analysis the direction is opposite). The standard solution to this problem is provided by the least-squares method, giving the *multilinear regression estimate*

$$\hat{\zeta}(k) = \left(\mathrm{E}\left\{\xi\xi^{\mathrm{T}}\right\}^{-1}\mathrm{E}\left\{\xi\zeta^{\mathrm{T}}\right\}\right)^{\mathrm{T}}\xi(k). \tag{6}$$

However, this estimate is typically not *robust* for high-dimensional data, as *collinearities* can cause the covariance matrix $\mathrm{E}\{\xi\xi^{\mathrm{T}}\}$ to become practically non-invertible. A simple fix to this problem is to add uncorrelated white noise to the data ξ; then the eigenvalues of the covariance matrix get farther from zero (this is closely related to *regularization* in the neural network algorithms). Thus, if the added white noise has covariance C, diagonal matrix with all positive entries, having always full rank, one has the (somewhat conservative) *ridge regression* formula

$$\hat{\zeta}(k) = \left(\left(C + \mathrm{E}\left\{ \xi\xi^{\mathrm{T}} \right\} \right)^{-1} \mathrm{E}\left\{ \xi\zeta^{\mathrm{T}} \right\} \right)^{\mathrm{T}} \xi(k). \tag{7}$$

When one selects $C = Q^{-1}$, $\xi = \bar{x}$, and $\zeta = u$ (or $\zeta = \bar{u}$) in (7), and when E is identified with $\mathcal{E}$, one can see the connection to formulas (3) and (4). Indeed, one can summarize the steady-state mappings in the following form with intriguing *dual symmetry*:

$$\begin{cases} \bar{x} = \phi^{\mathrm{T}}\,\bar{u} \\ \bar{x} = \varphi^{\mathrm{T}}\,u \\ \hat{u} = \phi\,\bar{x} \\ \hat{\bar{u}} = \varphi\,\bar{x}, \end{cases} \tag{8}$$

where the residual error is

$$\bar{u} = u - \hat{u}. \tag{9}$$

This all means that local level maximizations result in global level modeling. In the sense of information capture, the cybernetic model is the *best possible*:

– The feedforward section implements optimal (robust) modeling of the input data in terms of variance (enformation) preservation.
– The feedback implements optimal (robust) estimation (or "generative modeling") of the input data in terms of variance preservation.
– Thus, the closed loop with negative feedback implements optimal (robust) "statistical level control" of the input, or elimination of excitation from the environment.

Here, *optimality* in estimation is to be interpreted in the linear regression framework, and in modeling it means principal component (subspace) analysis perspective, in both cases meaning optimality in the statistical second moment sense. On the other hand, *robustness* in regression means reducing sensitivity to collinearity of variables; in the modeling part this robustness means pre-matching against candidate constructs, thus filtering noise. Briefly, as regression is enhanced through introduction of white noise, modeling is facilitated by the introduction of *black noise*.

The low-dimensional $\bar{x}$ is the *inner image* corresponding to the high-dimensional world state u, being filtered through the model characterized by the mapping φ. The system can "see" in its environment only those things that are already familiar to it!

3 Qualitative enhancements

When comparing the regularized regression formula (7) to the optimal formula (6), one can see that the additional term Q^{-1} in the "denominator" pushes the estimates towards smaller values. The regularized regression gives cautious (but robust) estimates; this means that not all modeled enformation in the environment can be exhausted by the system. Tuning up the coupling, increasing the coupling factors so that $q_i \to \infty$, does not help in enformation capture: even though the regression formulas become similar, the values of $\bar{u}$ go down with the tighter coupling. The increasing activity in the feedback loop only increases *dissipation* beyond a certain optimum; after $q_i = 4/\lambda_j$ the level of $\mathcal{E}\{\bar{x}_i^2\}$ indeed goes down.

No parameter tuning, for example, can solve this dissipation problem. To get rid of the regularizing matrix Q in the estimation formula, a *qualitative leap* is necessary.

It turns out that if the implicit feedback control structure is divided into separate modeling and estimation parts, and their combination is implemented as an *explicit feedback*, theoretically optimal least-squares estimation can be reached, and *all* of the available enformation can be captured on the system level. But more "intelligence" is then needed in the system. Indeed, *additional feedback loops* instead of the feedback through the environment are necessary to keep the system stable and to adjust the signals appropriately.

In the modeling part, an *internal* "social" feedback is needed from the system state $\bar{x}$. Assuming that the adaptation principles in the internal feedback are the same as in the original feedforward part, in steady state the additional negative feedback term looks like $-Q\mathcal{E}\{\bar{x}\bar{x}^{\mathrm{T}}\}\bar{x}$; when signals have found balance, the tensions compensate each other:

$$0 = Q\,\mathcal{E}\left\{\bar{x}u^{\mathrm{T}}\right\}u - Q\,\mathcal{E}\left\{\bar{x}\bar{x}^{\mathrm{T}}\right\}\bar{x}. \tag{10}$$

From this, one can solve the steady state:

$$\bar{x} = \mathcal{E}\left\{\bar{x}\bar{x}^{\mathrm{T}}\right\}^{-1}\mathcal{E}\left\{\bar{x}u^{\mathrm{T}}\right\}u = \Phi^{\mathrm{T}}u. \tag{11}$$

Here, the new symbol Φ is introduced; it stands for the asymptotical, optimal mappings. As in Part II, one can now derive

$$\Phi^{\mathrm{T}}\mathcal{E}\left\{uu^{\mathrm{T}}\right\}\Phi = \mathcal{E}\left\{\bar{x}\bar{x}^{\mathrm{T}}\right\} \tag{12}$$

and

$$\Phi^{\mathrm{T}}\Phi = I_n. \tag{13}$$

It can be shown that, again, this mapping spans the principal subspace of the data. The basis vectors are typically rotated, or $\mathcal{E}\{\bar{x}\bar{x}^{\mathrm{T}}\}$ is not diagonal. Explicit principal component analysis can be reached, for example, by explicitly masking the feedback matrix $Q\mathcal{E}\{\bar{x}\bar{x}^{\mathrm{T}}\}$ to become triangular, by zeroing the entries above (or below) the diagonal.

The learning principle is the same as before: the matrices contain the correlations among the incoming signal and the steady state — now the other "input" is the state itself. The difference between the roles of the signals is revealed by the minus sign: correlating states are not a resource but a burden, as the competing neighbors see each other as inhibitory “negative resources” that are to be avoided (compare to *anti-Hebbian learning* in artificial neural networks).

The feedback that is implemented by the system itself, instead of coming from the environment as a side-effect of, seems more “intelligent”, as it is not all about straightforward competition and starvation. But this also means that some level *understanding* is necessary: the actor has to see the wider view, observing the neighbors' actions and distinguishing it from the environment. One also has to see the original input u beyond the observer effect.

Correspondingly, to reach the optimal regression formula (6), one has to implement an internal negative feedback to have

$$\hat{u} = \mathcal{E}\left\{\bar{x}u^{\mathrm{T}}\right\}^{\mathrm{T}}\mathcal{E}\left\{\bar{x}\bar{x}^{\mathrm{T}}\right\}^{-1}\bar{x} = \Phi\,\bar{x}. \tag{14}$$

If the feedback loop is now closed, so that $\hat{u}$ is subtracted from u, *all* of the modeled enformation gets sucked out from the environment. The residual $\bar{u} = u - \hat{u}$ has pathological properties: $\bar{\lambda}_j = 0$ for all $1 \leq j \leq n$, so that the system is *completely coupled*.

When the presented control scheme has been introduced, *all* of the modeled enformation is inherited by the system variables $\bar{x}_i$ without having to waste it *at all* on dissipative structures (assuming that the internal feedback is lossless). If any of the subsystems has been capable of reaching such an invention, it has an evolutionary advantage — and, later, probably this strategy will dominate and characterize the whole system.

If some system has managed to reach that qualitative threshold towards lossless modeling and exploitation of its environment, the whole world will start changhing. Instead of being wasted in the processes, all modelled enformation cumulates in the system variables — and the other world can now see these enformation rich variables as *new resources*. This way, one can understand the emergence of *trophic layers* in different kinds of ecosystems. When enformation is not wasted, it can traverse in the chain (hierarchy) of systems a long way.

The entirety of systems becomes more complicated all the time and in all ways. Wherever there is unexploited enformation, there you will have finally have feedback control loops. The more intelligence a system has, the more creative the “system semiosis” can be. The end result os a *fractal structure of controllers*.

When there already exists a lot of structure in the world, the normality assumption of data does no more hold, and different controller building principles may become appropriate, instead of the neocybernetic strategy. The only thing that still remains is the principle of some kind of *recirculation*: in the unknown world, control always needs to be based on feedback, so that when the model is not yet perfect, the detected errors can be used for further refinement.

4 Why structures get more complex

In nature, even more complex systems develop on top of simpler ones, utilizing the enformation that is collected and concentrated on the lower levels. Why is it not so, as in the case of Miller–Urey experiment, that after a certain limit more complex compounds are too fragile and too improbable to exist in the bombardment of the hostile environment?

Indeed, in the enformation theoretic neocybernetic framework the ever increasing *complexification* can be motivated.

The entropy law states that everything decays towards states of increasing probability. And, normally, it is thought that emergence of new order fights against this principle, because it is extremely improbable that such structures would be constructed through random processes. Of course, the key here is that the processes are not random: the developments are directed towards enhanced enformation capture. The locally controlled pumps of enformation act like distributed, autonomous *Maxwell's demons*.

The neocybernetic system structure consists of the model of the environmental enformation. Because of the negative feedback, the modeled enformation is eliminated from the environment, becoming concentrated in the system, so that order emerges in the form of dynamic attractors. If all enformation is sucked out (this is of course the goal of the systems!), all variation vanishes, and, loosely speaking, the environment reaches the thermodynamic *heat death*. Specially, when the time axis is abstracted away, one can see that there clearly is entropy maximization: the model remains the same (thus being of "zero length") while eliminating enformation from the environment along the whole time axis.

This local enformation pumping takes place in all levels of the fractal fabric of systems, all systems utilizing the environmental resources; indeed, *entropy pursuit is the explanation for complexification*, rather than fighting against it. The physical systems and the living systems are perhaps not so different after all.

Whenever correlations are found in environmental data, models get constructed, and the corresponding controls suck that enformation out from the environment, leaving only noise there. Then the same procedure of Part I starts taking place in the remaining chaos; as long as there is noise, there is potential for enformation, the covariations there just waiting to be found.

Conceptually, there is now something important taking place: the point of view is changing. This far, the environment has been active, bombarding the system, whereas the system has been adapting in a humble manner; now, on the other hand, it is the controller that is in charge, the system modifying its environment (indeed, another view to entropy maximization is given by the whitening of the environmental data $\bar{u}$). — From now on, it is natural to look at the world "from above".

When the negative feedbacks change the environment to a more placid place, things are seemingly becoming "more comfortable" for the system — tragically, however, this is a fallacy; it is the *beginning of the end for the system*, as will be studied on Part IV. However, before that, another look at the controllers as forming the kernel of systems is taken; it is the *a priori* expectations that determine the interpretations, and to understand the *tragedy*, one needs to *feel sorry for systems*.

5 Monads as atoms of *life*

Despite the enhancements in evolutionary systems, one thing that remains the same is the (fractal) *control loop structure*. Such control loops can be seen as distributed "enformation pumps", stable attractors manifesting the properties and inbalances in the environment. A practical metaphor is to think of *whirls* in the entropy flow, in the very concrete Heraclitean spirit.

Where there is strong flow of enformation, nature implements *water wheels* — but sooner or later the free enformation flows are exhausted, and nature needs to implement *water pumps*.

Such local control chains, let us call them *monads*, are the "atoms" of the dynamic world in general, implementing some kind of *functions* in the system. The term monad was coined originally by Leibniz and the same connotations are appropriate here; however, now there are some differences. For example, the monads are now not eternal, and they are not always the same; they come and go depending whether there is enformation available. They are not the building blocks for everything there exists — only for the *living*. The living things are fractal collections of lower and higher level monads. The monads escape rough analyses: if the loops stop, their *essence* vanishes; only using the appropriate language you can "see" them.

This circular or cyclic nature can be observed at all levels in living systems, in all time scales. An example of the entropy-boosting long-term cycles is related to the question "which came first, the chicken or the egg?": all parts of the circular pattern of reproduction emerged together from non-existence, it is one example of the very basic continuums in life. Indeed, it is the cycles that are more fundamental in nature than the final manifestations of them, or the visible, temporary individuals.

These monads are an appropriate mental framework to study enformation theoretic systems in general — the fractal framework of *universal life*. Life is not limited to the biological world, as the same enformation pursuit takes place in mental, social, economical, etc., domains; and there are different *stages of being alive*.

Normally, "life" is characterized in terms of long lists of decriptive properties. However, it is the *eternal processes* that are the key thing, and the *population* is the real living object. Individuals are mainly needed to find new freedoms; reproduction is just the means for regeneration (see Part IV), and genetic codes (or other scriptures!) are needed to reconstruct the necessary monads in an orderly manner after a collapse.

When Stuart Kauffman proposed *autocatalytic sets* as the basis for the origin of life, he was almost right; however, without the concept of enformation, and without recognizing their control function, the loops alone will not work in a sensible way. Semantics has to be integrated in the analyses — *there is no life without the meaning of life*.

6 Towards "artificial life"

In sciences one finds models for the world. But only after the models are used for feedback, for exploiting the knowledge, there is evolutionary advantage: the new loops pump fresh enformation from the natural resources. Thus, *engineering work can be seen as implementing the latest, highest-level monads* in nature.

An industrial plant can be compared to a living cell: the difference is not qualitative but quantitative (however, in engineering systems it is the human who acts as a "signal carrier" and who determines the *system semiosis*). The "metabolism" has been optimized to implement the desired *function*. Gradually, the loops have been tailored to maintain the homeostasis ever better; the control structure is fractal, consisting of the stabilizing controls on the bottom, regulatory controls above them, and production optimization controls on top. The common goal of the controls is to make the enformation flow smooth and efficient, eliminating disturbances caused by the unknown and uncontrollable environment.

"Evolution" in industrial processes is carried out by man. Humans are needed to see the blockages in enformation flow and to eliminate them. The principles in engineering work are the same as they are in all evolution: *do not fix it if it is not broken*. Even though the structure were not optimal, radical changes are avoided. Creating new life from completely new "births" is so much more difficult than just augmenting a living system with new functionalities.

What makes things challenging, is that it is not only the technological systems that need to be taken care of; to keep processes up and running, also the *personnel* at the factory needs to be integrated in the system. And the mental models cannot easily be updated: for example, despite the control theoretical advances, simple unit controllers still rule in industry. The operation of the basic PID (or "Proportional–Inegrative–Derivative") controllers just is *intuitively understandable*.

Engineerirng work typically concentrates on polishing of details; however, new ways of thinking can make it possible to reach qualitative advances. For example, neocybernetic approaches seem to offer new tools for implementing *agent networks*: there would be applications in different kinds of sensor/actuator systems, like in *active vibration damping* or *adaptive power control* in distributed systems. The properties of neocybernetics assure self-controlled adaptation towards robustified structures, functional "eco-lockers" assuring sustainability also in changing environments.

In industrial plants one needs *operators* to implement the highest level feedbacks. Only humans can today interpret the visual information, for example, and apply it for control. However, now we have two complex interacting systems, the process itself and the *mind* that can assumedly both be modeled applying similar principles (see Part V); perhaps some kind of *intersubjectivity* can be implemented in models and used for automating the controls.

Applying the neocybernetic intuitions, it is possible to construct truly novel feedbacks on the levels not touched before. When the *parameters* in a process are seen as very slowly changing highest-level variables (see Part IV), one can start modeling interaction structures among them; assumedly one finds the familiar locally linear low-dimensional covariance structure there. In practice, one shuffles the process parameters, and detects the changes in *quality variables*; based on such models, "higher-level controllers" can be implemented. This idea has successfully been applied in complex plant simulators; see [1].

Neocybernetics and the related control intuitions open up still wider views. Traditionally one thinks that engineers carry out the final "dirty work", after the noble *understanding* has been provided in philosophy, in mathematics, and in natural sciences. Now things get inverted: if one looks at the evolutionary developments in nature as a story told in the syntax of mathematics, it is natural sciences that provide the semantics for the language; but it is the *control engineering intuition* that is needed to understand the *narrative*. — How the drama in the presented framework becomes completed (or the "cycle from chaos to chaos") — this is explained in Part IV.

References

1. *Neocybernetics — Pragmatic Semiosis by Complex Adaptive Systems.* Research pages accessible in Internet through http://neocybernetics.com.