Elastic Systems: Another View at Complexity

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Abstract

Elastic systems study complex systems in a cybernetic perspective, assuming that it is the internal feedbacks and interactions that are responsible for the observed functionalities. What is more, in the *neocybernetic* framework there are some additional assumptions — like *balance* and *linearity pursuit* — that make it possible to apply efficient theoretical tools for model construction. It turns out that the very simple starting points are enough to assure *self-regulation* and *self-organization.* This self-organization can be interpreted in terms of multivariate statistics: The system implements *principal subspace compression* of the input resources. This optimality makes it possible to make evolutionary hypotheses, and generalize the models to different phenospheres.

1 Introduction

Stephen Wolfram (Wolfram, 2002) has said it most explicitly: The era of traditional science is over, mathematics cannot give added value to analyses of real-life complex systems. Do we have to be content with weaker descriptions of nature, regressing to "postmodern" ways of doing science (Horgan, 1997)?

Whereas there is motivation for such claims — analyzability truly collapses in powerful enough systems — this is not necessarily a deadlock. For example, Wolfram's *cellular automata* are just another model, and the problems he discusses are problems of that model family.

What if real systems are not fundamentally that complex? Indeed, *neocybernetics* is an approach to complex systems where it is explicitly assumed that modeling still *is* possible. Whereas there are typically many approaches to constructing models, this neocybernetic starting point gives strong guidelines in which directions the modeling has to proceed. There are various assumptions that are somewhat counterintuitive and in contrast with the mainstream complexity theories (these are explained in depth in (Hyötyniemi, 2006)):

- **Universality pursuit.** To make the models scalable beyond "toy worlds", the model structure has to be *linear.*
- **Emergence support.** To make it possible for new structures to emerge, environment-orientedness and *emphasis on data* is needed.
- **Statistical relevance.** To make the observation data representative, the models have to be stationary and *stable* (in the large).
- **Practical plausibility.** The models have to be based on local operations only, as there exist *no structures of centralized control* in nature.

Complexity intuition says that to find interesting behaviors, the models *must* be nonlinear, and they *must not* be in any equilibrium. Similar intuitions apply also to traditional cybernetics (Wiener, 1948). However, in neocybernetics the loss in expressional power is compensated by high dimensionality and dynamic nature of the models. And the homeostasis assumption[1] does not mean that the system would be dead: It has to be kept in mind that one is studying dynamic equilibria rather than static balances, the key functionalities being provided by internal tensions — interaction structures and feedbacks. The emphasis on statistic phenomena means that general attractors rather than individual processes are concentrated on. Strong conceptual tools are needed to maintain the cumulation of complexity in such models, and to give a firm basis for studying *emergence.* Such tools are available in mathematics: Linear algebra with matrix calculus, and system theory with control engineering.

What is the relevance of such discussions to artificial intelligence? It needs to be kept in mind that

[1] Of all possible models, unstable ones are much more common. Neocybernetics does not try to cover all mathematically possible models, only the physically meaningful ones — such that have *survived,* being capable of reaching balance in their environments

cybernetics was one of the cornerstones of early AI, even though the methodologies soon grew apart. It seems that in such mathematical framework some of the age-old AI dilemmas can be attacked: One can find a connection between numeric data and emergent symbolic structures. It can be claimed that *qualitative constructs are attractors of quantitative dynamic processes.*

2 Interpretations of models

Mathematics is a strong language, and it can efficiently be used for "discussing" about new concepts and intuitions. As an example of this, it is shown how the same model can be seen in different ways.

2.1 Towards elastic view of systems

Coupling variables together by introducing a set of constraints is a very general way to define a (linear) model:

$$0 = \Gamma z. \tag{1}$$

Assume that the variables in z are divided in two parts: Vector u, dimension m, describes the environmental conditions, whereas vector $\bar{x}$, dimension n, contains the system-specific *internal variables*, somehow characterizing the equilibrium state of the system. Rewriting the constraints characterizing the system, one can distinguish between the variables:

$$A\bar{x} = Bu. \tag{2}$$

It is assumed that there are as many constraints here as there are latent variables, so that A is square. Because of environment-orientedness, the internal variables are assumed to be directly determined by the environment, so that there assumedly is a (linear) dependency between $\bar{x}$ and u. Assuming that A is invertible, one can explicitly solve the unique linear function from the environmental variables into the system state:

$$\bar{x} = A^{-1}B\,u, \tag{3}$$

so that one can define an explicit mapping matrix from u to $\bar{x}$ as $\phi^T = A^{-1}B.$. However, the main motivation for the formulation in (2) is that one can formally extend the static model into a dynamic one.

The formula (2) only characterizes the final visible global balance in the system, but one has to remember that it is local operations only that exist — how can such uncoordinated local actions implement the global-level behaviors? Indeed, one needs to extend studies beyond the final balance, and take into account the dynamic behaviors caused by the inbalances.

Formula (2) can be interpreted as a balance of tensions determined by forces $A\bar{x}$ and Bu, caused by the system itself and by the environment, respectively. If the forces are not in balance, there is a drift:

$$\frac{d\,x}{\gamma\,d\tau} = -Ax + Bu. \tag{4}$$

The parameter γ can be used for adjusting the time axis. The first term in (4) introduces *feedback.* The steady state equals that of (3), so that $\lim_{\tau\rightarrow\infty} x = \bar{x}$ for constant u. Because of linearity, this steady state is unique, no matter what was the initial state. Using the above construction, the static pattern has been transformed into a dynamic pattern — the observed equivalences are just emergent phenomena reflecting the underlying dynamic equilibrium.

How can such a genuine extension from a static model into a dynamic one be justified? It needs to be observed that there *must* exist such a inner structure beyond the surface. The seemingly static dependencies of the form (1) have to be basically dynamic equilibria systems so that the equality can be restored after disturbances: The local actors, whatever they are, do not know the "big picture", and it is the interactions among the actors that provide for the tensions resulting in the tendency towards balance.

What causes the dynamics? Thinking of the mindless actors in the system, the only reasonable explanation for the distributed behaviors is some kind of *diffusion.* It is some kind of gradients that only are visible at the local scale. So, interpreting (4) as a (negative) gradient, there has to exist an integral — a criterion that is being minimized. By integration with respect to the variable x, it is found that

$$\mathcal{J}(x,u) = \frac{1}{2}x^TAx - x^TBu \tag{5}$$

gives a mathematical "pattern" that characterizes the system in a yet another way in a global scale.

For a moment, assume that vector u denotes *forces* acting in a (discretized) mechanical system, and x denotes the resulting *deformations.* Further, assume that A is interpreted as the *elasticity matrix* and B is *projection matrix* mapping the forces onto the deformation axes. Then, it turns out that (5) is the difference between the *internal* and *external potential energies* stored in the mechanical system. Principle of minimum potential (deformation) energy (*et al*, 2001) states that a structure under pressure ends in minimum of this criterion, trying to exhaust the external force with minimum of internal deformations.

However, the formula (1), and thus (5), can be seen to characterize a wide variety of complex balance systems, not only mechanical ones. This means that in non-mechanical cybernetic systems, the same intuition concerning understanding of mechanical systems can be exploited. It does not matter what is the domain, and what is the physical interpretation of the "forces" u and of the "deformations" $\bar{x}$, the structure of the system behavior remains intact: As the system is "pressed", it yields in a more or less humble manner, but when the pressure is released, the original state is restored. Applying these intuitions, one can generally speak of *elastic systems.*

2.2 Pattern matching

There is yet another perspective to see the global pattern that is beneficial because it introduces explicit feedback between the system and its environment.

What comes to the tensions caused by the cost criterion (5), it is equivalent to another criterion:

$$J(x,u) = \frac{1}{2}\,(u-\varphi x)^T\, W\,(u-\varphi x)\,. \qquad (6)$$

This formulation represents a *pattern matching* problem, where one tries to minimize the difference between the "environmental pattern" u and its reconstruction using *features* φ_i weighted by the variables x_i. The correspondence between the cost criteria is reached when one defines the matrices as

$$\left\{ \begin{array}{lcl} A & = & \varphi^T W \varphi \\ B & = & \varphi^T W. \end{array} \right. \qquad (7)$$

Criterion (6) gives another view too see the same gradient-based minimization (4). When (6) is minimized using the steepest descent gradient approach, the continuous-time process implementing this minimization is

$$\frac{d\,x}{\gamma\,d\tau} = \varphi^T W\,(u-\varphi x)\,. \qquad (8)$$

It is the gradients that can only be seen by the local actors in a system. From now on, assume the mathematical gradient formulation (8) captures the essence of the natural dynamics, too. Whereas the matrix ϕ^T implements a mapping from the environmental variables u into the system variables $\bar{x}$, the feature matrix φ can be interpreted as an inverse mapping from the space of x back into the space of u. This can be seen as a feedback loop.

However, such observations above have limited value if the data structures ϕ, φ, and W (or A and B) cannot be determined. To attack this problem, a wider perspective is needed.

3 Role of feedback

The main functionality in a cybernetic system comes from the feedback structures. As shown below, there is no need for some "master mind" to implement such interaction structures.

3.1 Exploiting non-ideality

There are no unidirectional effects in real systems: Information flows cannot exist without physical flows that implement them. When energy is being consumed by the system, this energy is taken from the environment, or environmental "resources" are exhausted. To understand these mechanisms, study the pattern matching process (8): There is $-\varphi x$ defining some kind of negative feedback structure, representing real material flow from the system into the environment, denoting the resources being exhausted. The changed environment becomes

$$\tilde{u} = \underbrace{u}_{\substack{\text{actual}\\ \text{environment}}} - \underbrace{\varphi x}_{\text{feedback}}\,. \qquad (9)$$

The system never sees the original u but only the distorted $\tilde{u}$, where the momentary consumption by the system, or φx, is taken into account. Clearly, as the environment affects the system and the system affects the environment, there exists a dynamic structure; again, one is interested in the final balance after transients:

$$\bar{u} = u - \varphi\bar{x}. \qquad (10)$$

Employing this actual, observable environment, one can redefine the mapping ϕ^T so that

$$\bar{x} = \bar{\phi}^T \bar{u}. \qquad (11)$$

When studying the steady state, there is efficiently an algebraic loop in the system, and this means that this structure has peculiar properties. Multiplying (10) from the right by $\bar{x}^T$, taking expectations, and reordering the terms, one receives

$$\mathrm{E}\{(u-\bar{u})\bar{x}^T\}\mathrm{E}\{\bar{x}\bar{x}^T\}^{-1} = \varphi, \qquad (12)$$

so that, when one defines a quantity for measuring the discrepancy between the undisturbed open-loop environment and the disturbed closed-loop environment,

$$\Delta u = u - \bar{u}, \qquad (13)$$

the expression (10) can be written in the form

$$\Delta u = \mathrm{E}\{\bar{x}\Delta u^T\}^T \mathrm{E}\{\bar{x}\bar{x}^T\}^{-1}\,\bar{x}. \qquad (14)$$

Variables in $\bar{x}$ and Δu are mutually connected, they vary hand in hand, but together representing the same mapping as φ. Indeed, this Δu can be seen as a "loop invariant" that helps to see properties of the feedback loop, and it turns out to offer a way to reach simplified analysis of the signals. Because Δu assumedly linearly dependent of u, one can interpret this variable as the actual input driving the whole loop, so that there exists a mapping Φ^T

$$\bar{x} = \Phi^T \, \Delta u. \tag{15}$$

Assuming that the feedback can implement stabilization, the system will search a balance so that

$$\bar{x} = \Phi^T \varphi \, \bar{x}. \tag{16}$$

To have not only trivial solutions (meaning $\bar{x} \equiv 0$), there must hold

$$\Phi^T \varphi = I_n, \tag{17}$$

so that the feedforward and feedback mappings have to be mutually orthogonal. This is a very stringent constraint, and it essentially determines the properties of the feedforward matrix Φ. It turns out that a symmetric formulation is valid:

$$\bar{x} = \mathrm{E}\{\bar{x}\bar{x}^T\}^{-1} \mathrm{E}\{\bar{x}\Delta u^T\} \, \Delta u. \tag{18}$$

This expression connects $\bar{x}$ and Δu with their statistical properties, making it possible to combine two time scales, or emergent levels.

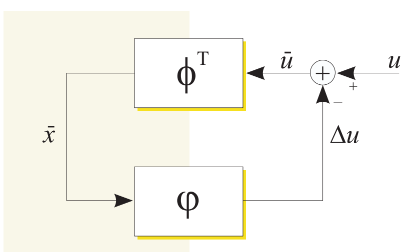

Figure 1: Feedback structure between the system and environment

3.2 Constraints vs. freedoms

Above, the balances of x were studied as the environment u was assumed fixed. However, to reach interesting results, the neocybernetic principles need to be exploited again: It is assumed that there exist various levels of seeing the system, and at each of the levels, the balances are exploited. Whereas u was assumed to remain constant this far, it only has much slower dynamics than x, and on the wider scale, the environment changes. But assuming stationarity of the environment, or balance on the higher scale, so that u has fixed statistical properties, one can find a "balance model of balances". A truly cybernetic model is a *second-order balance model*, or a *higher-order balance model* over the variations in the system — at these levels beyond the trivial first level balance, one can reach stronger views to see the systems, including *self-organization,* as shown below.

So, assume that dynamics of u is essentially slower than that of x and study the statistical properties over the range of $\bar{x}$, and, specially, construct the covariance matrix of it. One can show that there holds

$$\left(\Phi^T \mathrm{E}\left\{\Delta u \Delta u^T\right\} \Phi\right)^3 = \Phi^T \mathrm{E}\left\{\Delta u \Delta u^T\right\}^3 \Phi. \tag{19}$$

If $n = m$, any orthogonal matrix $\Phi^T = \Phi^{-1}$ will do; however, if $n < m$, so that x is lower-dimensional than u, the solution to the above expression is nontrivial: It turns our that *any subset of the principal component axes of the data* Δu *can be selected to constitute* Φ, that is, the columns Φ_i can be any n of the m covariance matrix eigenvectors θ_j of this data. Further, these basis vectors can be mixed, so that $\Phi = \theta D$, where D is any orthogonal $n \times n$ matrix, so that $D^T = D^{-1}$.

These results show that any set of covariance matrix eigenvectors can be selected in Φ. However, in practice it is not whatever combination of vectors θ_j that can be selected: Some solutions are *unstable* when applying the iterative adaptation strategies. Indeed, the only stable and thus relevant solution is such where it is the n most significant eigenvectors (as revealed by the corresponding eigenvalues) that constitute the matrix Φ in convergent systems. This means that the system implements *principal subspace analysis* for input data. Because of the mixing matrix D, the result is not unique in the sense of principal components, but the subspace spanned by them is identical, and exactly the same amount of input data variation is captured.

The properties of principal components are discussed, for example, in (Basilevsky, 1994). It turns out that the directions of *maximum variation* are captured (see Fig. 2). If it is assumed that (co)variation structures in data carry information, it can be claimed that compression of data applying PCA maximally captures the information available.

It turns out that the neocybernetic view of looking at a system is *opposite* to the traditional one: Whereas in (1) one concentrates on *constraints* in the data space, now it is *degrees of freedom* that are concentrated on. This approach is well suited for model-

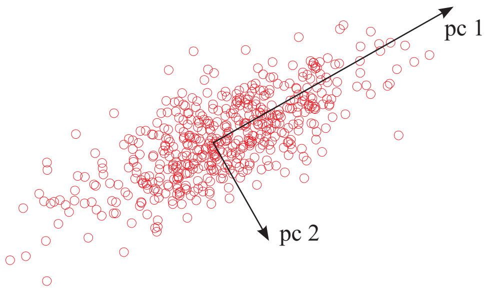

Figure 2: How the PCA basis is oriented towards information — as defined in a technical way

ing of a complex system: There exist assumedly huge numbers of (unknown) feedback structures in a cybernetic system keeping the system in balance; each of these constraints binds system variables closer together, and as a result, there typically remain just a few of the directions in the high-dimensional data space that are non-fixed. The high number of constraints changes into a low number of degrees of freedom — and, applying *Ockham's razor,* a simpler model is *better.*

Now one can conclude that completely local operations result in non-trivial structures that are meaningful on the global scale: Competitive learning without any structural constraints results in self-regulation (balance) and self-organization (in terms of principal subspace). Feedback through the environment, or competition for the resources, results in stabilization and organization of the system.

4 Towards second-order balance

The PCA subspace is determined by the columns φ_i — the above results are of limited use if there is no adaptation in the system. Theodosius Dobzhansky's intuition can be extended beyond biological domains: *No model of a complex system makes sense without reference to evolution.*

4.1 Emergy and evolutionary fitness

The effect of the environmental pressures on an elastic system can be easily quantified: Just as in the case of a potential field, it is the product of the force and displacement that determines the change in potential energy. Similarly, regardless of the physical units of the variables, one can interpret the product $\bar{x}_i\bar{u}_j$ in terms of *energy (power) transferred from the environment into the system* through the pair of variables $\bar{u}_j$ and x_i. This concept deserves a name: In what follows, this "emergent level energy" is studied along the following definition:

> **Emergy** (a scalar dimensionless quantity) is the product of the (abstract) force and the corresponding (abstract) deformation.

As it turns out, this emergy is "information energy" that is the prerequisite for emergence of information structures.

When trying to determine how the system is to adapt, the key challenge is to determine the *goals of evolution.* In natural systems there are no explicit goals as determined from outside: Neocybernetic environment-orientedness suggests a criterion emphasizing some kind of *match with environment.* Indeed, applying the above discussion concerning energy/power transfer from the environment into the system and back, an intuitively appealing fitness criterion would be

> Maximize the average amount of emergy that is being transferred between the system and the environment.

No matter what is the physical manifestation of the environmental variables, a surviving system interprets them as resources, and exploits them as efficiently as possible. However, it must be remembered that there is not only the effect from the external environment into the internal system — there is a symmetric two-way interaction that takes place. It is the matrices $\bar{\phi}^T$ and φ that characterize the emergy transfer between the system and its environment. It is not only so that $\bar{u}$ should be seen as the "force" and $\bar{x}$ as the effect: $\bar{x}$ can be seen as the action and $\bar{u}$ as the reaction just as well.

The momentary emergy traversing from the environmental variable j to the state variable i can be written as $\bar{x}_i\bar{u}_j$, or, when written in a matrix form simultaneously for all variables, $\bar{x}\bar{u}^T$. Similarly, the momentary emergy traversing from the state variable i to the environmental variable j can be written as $\bar{u}_j\bar{x}_i$, or, when written simultaneously for all variables, $\bar{u}\bar{x}^T$. If evolution proceeds in a consistent manner, the differences among such variable pairs should determine the growth rates of the corresponding links between the variables; when the mapping matrices $\bar{\phi}^T$ and φ are defined as shown above, one can assume that a stochastic adaptation process takes place, the observations of prevailing variable levels determining the stochastic gradient direction:

$$\begin{cases} \dfrac{d\,\bar{\phi}^T}{dt} & \propto & \bar{x}(t)\bar{u}^T(t) \\ \dfrac{d\,\varphi}{dt} & \propto & \bar{u}(t)\bar{x}^T(t). \end{cases} \quad (20)$$

Because of the feedback through the environment, $\bar{x}$ and $\bar{u}$ remain bounded, and it is reasonable to assume that the processes (20) find a fixed state, or statistical *second-order balance.* The solution for this fixed state can be assumed to be such that the matrix elements $\bar{\phi}_{ji}$ are relative to the correlations between $\bar{x}_i$ and $\bar{u}_j$, or

$$\bar{\phi}^T = q\,\mathrm{E}\left\{\bar{x}\bar{u}^T\right\}, \quad (21)$$

and in the backward direction,

$$\varphi = b\,\mathrm{E}\left\{\bar{u}\bar{x}^T\right\}. \quad (22)$$

This means that the matrices $\bar{\phi}$ and φ should become proportional to each other:

$$\varphi = \frac{b}{q}\,\bar{\phi}. \quad (23)$$

Here, the parameters q and b are *coupling coefficients:* For example, it turns out that q can be interpreted as *stiffness.* As it turns out, these factors scale the signal levels in the system and in the environment.

It needs to be recognized that the adaptation in the system according to (21) is completely local for any element in the matrices $\bar{\phi}$ and φ even though the assumed goal of the evolutionary process is presented in a collective matrix format. As it turns out, the net effect is, however, that the adaptation principle (20) makes φ span the principal subspace of the original input u, and also the subspace determined by Φ becomes aligned.

4.2 Equalization of stiffnesses

The signals $\bar{x}$ and $\bar{u}$, as connected by $\bar{x} = \bar{\phi}^T\bar{u}$, have peculiar properties. For example, multiplying the expression from the right by $\bar{x}^T$ and taking expectation, one has an expression for the latent vector covariance:

$$\mathrm{E}\{\bar{x}\bar{x}^T\} = q\,\mathrm{E}\{\bar{x}\bar{u}^T\}\mathrm{E}\{\bar{x}\bar{u}^T\}^T. \quad (24)$$

This holds *if* the latent variables x_i do not fade away altogether. On the other hand, multiplying the expression from the right by $\bar{u}^T$ and taking expectation, one has

$$\mathrm{E}\{\bar{x}\bar{u}^T\} = q\,\mathrm{E}\{\bar{x}\bar{u}^T\}\mathrm{E}\{\bar{u}\bar{u}^T\}. \quad (25)$$

Substituting this in (24),

$$\mathrm{E}\{\bar{x}\bar{x}^T\} = q^2\,\mathrm{E}\{\bar{x}\bar{u}^T\}\mathrm{E}\{\bar{u}\bar{u}^T\}\mathrm{E}\{\bar{x}\bar{u}^T\}^T, \quad (26)$$

or

$$\frac{1}{q}\,I_n = \bar{\theta}'^T\,\mathrm{E}\{\bar{u}\bar{u}^T\}\,\bar{\theta}', \quad (27)$$

where

$$\bar{\theta}'^T = \sqrt{q}\,\mathrm{E}\{\bar{x}\bar{x}^T\}^{-1/2}\mathrm{E}\{\bar{x}\bar{u}^T\}. \quad (28)$$

From (24), it is evident that there holds

$$\bar{\theta}'^T\bar{\theta}' = I_n. \quad (29)$$

The results (27) and (29) mean that the columns in $\bar{\theta}'$ span the subspace determined by n of the principal components of $\bar{u}$, so that $\bar{\theta}' = \bar{\theta}D$, where $\bar{\theta}$ is a matrix containing n of the covariance matrix eigenvectors, and D is some orthogonal matrix. All eigenvalues $\bar{\lambda}_j$ in the *closed loop* equal $1/q$. This peculiar equalization of system variances is visualized in Fig. 3. When q grows, system *stiffness* increases, and Δu becomes closer to u.

Assume that the coupling coefficients q_i vary between latent variables, so that one has $\bar{\phi}^T = Q\mathrm{E}\{\bar{x}\bar{u}^T\}$ for some diagonal coupling matrix Q. Following the above guidelines, it is easy to see that the matrix of eigenvalues for $\mathrm{E}\{\bar{u}\bar{u}^T\}$ becomes Q^{-1}. What is more interesting, is that one can derive for the symmetric matrix $\mathrm{E}\{\bar{x}\bar{x}^T\}$ two expressions: Simultaneously there holds $\mathrm{E}\{\bar{x}\bar{x}^T\} = Q\mathrm{E}\{\bar{x}\bar{u}^T\}\mathrm{E}\{\bar{x}\bar{u}^T\}^T$ and $\mathrm{E}\{\bar{x}\bar{x}^T\} = \mathrm{E}\{\bar{x}\bar{u}^T\}\mathrm{E}\{\bar{x}\bar{u}^T\}^T Q$. For non-trivial Q this can only hold if latent vector covariance is diagonal; what is more, the vectors in $\bar{\theta}^T = \sqrt{Q}\mathrm{E}\{\bar{x}\bar{x}^T\}^{-1/2}\mathrm{E}\{\bar{x}\bar{u}^T\}$ now not only span the principal subspace, but they are the PCA basis vectors themselves (basis vectors not necessarily ordered in the order of significance). This means that the modes become separated from each other if they are coupled to the environment in different degrees.

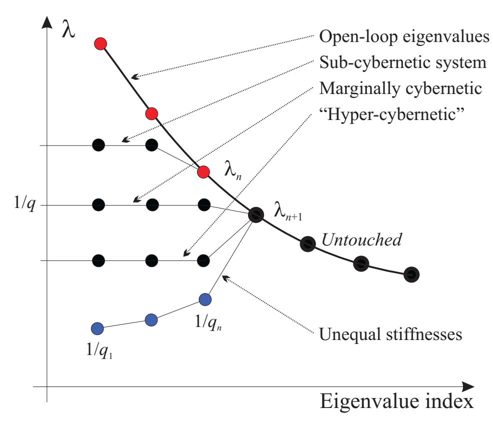

Figure 3: How the open-loop variances are modified by the elastic evolution

5 Elasticity intuitions

Why the above framework was selected, and why the tedious derivations were carried out? As shown below, it can be claimed that the framework of elastic systems makes it possible to reach the age-old goal of complexity theory: Complex systems in very different phenospheres have *the same model.*

5.1 Modeling of populations

The elasticity intuitions and neocybernetic model structures can be applied in artificial intelligence (modeling of signals in infosphere) as well as in artificial life (modeling of signals in biosphere). Perhaps the ideas can also be extended to real intelligence and real life. Here, the AL perspective is first elaborated on.

When modeling some real systems, like populations, it is, in principle, easy to reinterpret the symbols: The vector $\bar{x}$ represents *population sizes,* $\bar{u}$ is the vector of *available resources* (different types of food and other living conditions), and matrices A and B contain the *interaction factors* (competition) among populations. The columns in ϕ can be called *forage profiles,* exploitation conventions corresponding to the populations. But is there such universality among complex systems?

There can exist many alternative ways to survive in the environment — why should one favor one specific model structure? The key point is that *following the neocybernetic model, there is evolutionary advantage.* When the coupling with the environment increases, in a system obeying the neocybernetic model *optimality in terms of resource usage is reached.* In the long run, it is the models that implement the PSA model that can best be matched against variations in the resources $\bar{u}$ (in terms of quadratic variation criteria), resulting in most efficient exploitation of the resources. And populations with optimal strategies assumedly outperform others in terms of biomass and more probable survival. As no other locally controlled model families exhibit the same functionality, *succesfully competing populations assumedly must have adopted neocybernetic strategy.*

It is perhaps hard to believe that the very nonlinear genetic mutations and accomodation processes, etc., would have anything in common with the cellular adaptation details. How could the same model apply? The key observation here is that it is, again, only the dynamic equilibria that are studied, not the all possible routes there. Whereas the adaptation processes can be very complicated and varied, the final emergent optimum can be unique in terms of tensions

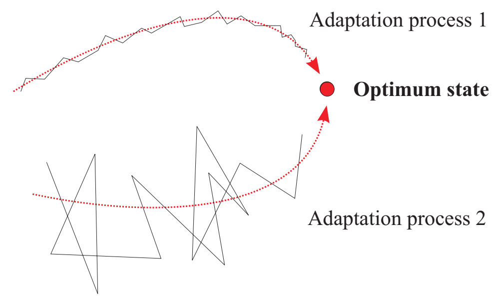

Figure 4: The adaptation strategies and dynamics can be very different in different kinds of systems — but the final state, the hypothetical optimum, is the same for all

(see Fig. 4). When concentrating on the balance only, it is also the dimensionality of the problem that goes down, making the analysis feasible.

Traditionally, ecological models concentrate only on a single species or interactions between two species (for example, see Turchin (2003)). Larger models try to characterize the *niches,* implementing explicit *forage profiles* that describe the resource specifications for each species. However, such models for complete ecologies need careful tuning; evolutionary strategies typically become unstable, meaning that most of the species become extinct, only some of them prospering and exhausting all resources.

When applying the neocybernetic model, ecosystem simulations remain stable even though the dynamics looks "naturally chaotic": There exists unforced dynamics in different time scales. As the adaptation in the system is based on cybernetic evolution, there is vivid dynamics, but no explosions take place. No fine tuning is needed: If there is enough variation in the resources, after system-wide adaptation a balance is found where there is a "niche" for every species. The niches are characterized by the principal subspace dimensions, the forage profiles ϕ_i mapping from the prevailing resource vector $\bar{u}$ to the balance population $\bar{x}_i$. The roles of the species cannot be predicted, only the subspace that is spanned by all of them together is determined by the environment. The above key observations concerning the neocybernetic model properties can be summarized:

- **Robustness.** In nature, no catastrophic effects typically take place; even key species are substituted if they become extinct, after a somewhat turbulent period. Using the neocybernetic model, this can also be explained in terms of the principal subspace: If the profiles are almost orthogonal, in the spirit of PCA, changes in some of the latent variables are independent of each other, and disturbances do not cumulate. Also because of the principal subspace, the system reacts fast to *relevant* changes in the environment, whereas sensitivity towards random variations that are not supported by the long-term signal properties are suppressed.

- **Biodiversity.** In nature, there are many competing species, none of them becoming extinct; modeling this phenomenon seems to be extremely difficult. Now, again, this results from the principal subspace nature of the model: As long as there exist various degrees of freedom in input, there is room for different populations. Within species, this also explains why in balance there exists variation within populations as the lesser principal components also exist.

Such populations can reside just as well in economies as in ecologies, and the models are applicable for social structures within populations. When relaxing the variables, also *memetic systems* can be studied can be studied applying the same intuitions as trying to do "pattern matching" in the ideasphere. In technical network designs the cybernetic models can help to reach natural-like robustness.

5.2 Power of analogues

When applying linear models, the number of available structures is rather limited – indeed, there exist more systems than there are models. This idea has been applied routinely: Complicated systems are visualized in terms of more familiar systems with the same dynamics. In the presence of modern simulation tools, this kind of lumped parameter simplifications seem somewhat outdated — however, in the case of really complicated, poorly structured distributed parameter systems, such analogies may have reincarnation.

Another class of analogues in addition to the mechanical ones can also be constructed: One can select *electrical current* and *voltage* rather than force and deformation. The external forces change to electrical loads disturbing the system: The deformation is the voltage drop, and the compensating action is the increased current supply (or vice versa).

The electric analogy makes it possible to extend the inner-system discussions onto the environmental level, to inter-system studies. When there are many connected systems interacting, one subsystem exhausting emergy supplied by the other subsystems — or providing emergy for the others, or transferring emergy between them — the criterion for system fitness can be based on the power transmission capabilities among the systems. It is the product of current and voltage that has the unit of power, so that exactly the above discussions apply. Only the intuitions change: Now one can utilize the *inter-system* understanding supplied by electrical systems. Remember that the maximum throughput without "ringing" between electrical systems is reached when there is *impedance matching:* The output impedance in the former system and the input impedance of the latter one should be equal, otherwise not all of the power goes through but bounces back. This same bouncing metaphor can be applied efficiently also in nonelectrical environments — the variables can have different interpretations but the qualitative behaviors remain the same. It is the coupling factors q_i among interacting systems that should be matched by evolution.

6 Conclusion

It can be concluded that elastic systems seem to offer a promising framework for modeling of different kinds of distributed agents. No explicit communication among the agents is needed — higher-level structure emerges when interactions take place through the environment. These claims are best illustrated using practical examples: Such examples of application of elasticity thinking are presented elsewhere in this Proceedings.

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