Session 6

Networks of Agents

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It seems that in today's world everything is linked – or networked – together. When studied closer, networks varying from internet to a coctail party or ancient bacteria resemble each other and the same rules apply. However, there are no solid tools for studying networks. That is why we have to learn how to interpret these nets and to try to see if "network thinking" could enable us to see something new or at least see things in a different light.

6.1 Introduction

In February 7^{th} 2000 at 10:20 Pacific Standard Time, Yahoo — one of the most popular search engines at that time — started to receive billions of service requests. This would have been a celebrated event but there was a problem, however. The requests were not produced by ordinary people trying to find information. They were produced by computer program running in a large number of computers. The request was not very fruitful either, the messages sent to hundreds of computers in Yahoo's Santa Clara, California, headquarters contained only the message "Yes, I heard you!" [24]. Yahoo's computers were under a "Denial Of Service" attack while millions of legitimate customers, who wanted a movie title or an airline ticket, waited. Next day the same attack was targeted at Amazon.com, eBay, CNN.com, ETrade and Excite.

The damage caused to these "giants of the Web" was huge and thus a very high-profile search was launched by the Federal Bureau of Investigation (FBI). The common opinion was that the attack must be doings of a group of sophisticated crackers who had hijacked hundreds of computers in schools, research labs and businesses and turned them into zombies screaming "Yes, I heard you!" to Yahoo and the others.

Finally the FBI solved the crime. They did not find the much-anticipated cyberterrorist organization, however. Instead, they tracked down a single fifteen year old teenager living in the Canadian suburbs. Funny thing was that the FBI would have never found the boy hadn't he been bragging with his doings in a chat room where he called himself *MafiaBoy*.

The *MafiaBoy* successfully managed to halt the operation of billion-dollar companies with access to best computer security experts in the world. Although, it was noticed that *MafiaBoy* was not even very skilled among computers he still managed to organize this attack from his suburban home using his modest desktop computer. And what is more interesting is that these big companies were completely powerless against this type of rudimentary attack.

What made it possible for this fifteen–year–old to cause this kind of damage? The answer lies in the structure of the complex network that he was using to accomplish his task. Although it's clear that he could not have done anything to these computers directly, he utilized the properties and weaknesses of the underlying network — the Internet. If a mere youth can cause this kind of harm, what could a small group of trained and skilled professionals achieve? How vulnerable are we to such attacks?

In the light of the example above, it seems clear that the different properties and the weaknesses of the Internet have to be studied carefully in order to assure reliable operation of legitimate companies and other every–day–activities that we use it for. However, there have been findings suggesting that the same kind of structures and behaviors can be found in a number of different kinds of networks varying from a social network in a cocktail party to metabolic network within a cell.

Thus, a growing number of researchers believe that there must be some universal laws describing these nets and once we find them for a certain network we can possibly utilize the laws in some other nets and explain the behavior more precisely.

6.1.1 Reductionism

Reductionism has been very popular method among scientists in the last decade. The idea behind reductionism is that to comprehend nature we have to study its smallest parts. Once we understand the parts, it will be easy to grasp the whole. Divide and conquer; the devil is in the details. To understand the universe we have to study atoms and superstrings, molecules to comprehend life, individual genes to understand complex human behavior and so on.

However, even if we know almost everything there is to know about the pieces we are still as far as we have ever been from understanding the nature as a whole. The main problem with reductionism is that once we start to put our little pieces of the puzzle together, we run into the *hard wall of complexity*. There are billions of ways to do this assembly, if you will, and we would need something more than just the knowledge of the pieces. One possible remedy to this problem is networks, they are everywhere. All we need is an eye for them. Once the general laws governing the properties and formation of different networks are studied and understood, we may have a very powerful *"roadmap"* or *"blueprint"* describing a variety of our complex problems.

6.2 Advent of Graph Theory

Graph theory can be considered the basis for today's thinking about networks. It was born in 1736 when Leonhard Euler, a Swiss born mathematician who spent most of his career in Berlin and St. Petersburg, offered a proof to a problem that had been troubling the people of Königsberg [8].

6.2.1 Königsberg bridges

At Euler's time Königsberg was a flowering city in eastern Prussia since it had not yet confronted the horrors of the Second World War. The city had a busy fleet of ships and their trade offered a comfortable life to the local merchants and their families. Due to good economical situation the city officials had built seven bridges across the Pregel river. The bridges can be seen in the Figure 6.1, where the seven bridges connect four pieces of land (A, B, C and D) together. For a long time there was an amusing mind puzzle among the citizens of Königsberg: "Can one walk across the seven bridges and never cross the same one twice?"

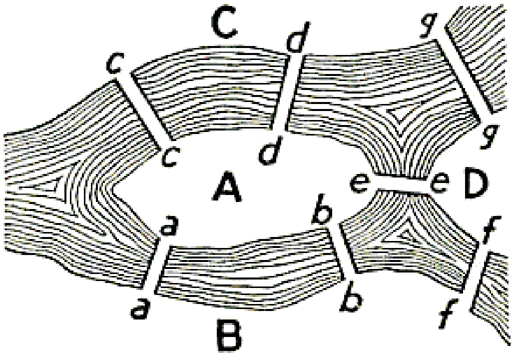

Figure 6.1: Königsberg bridges

Once Euler heard about this he was intrigued about the problem and decided to search for a solution. This he quickly found and was able to proof that such a path does not exist. He even wrote a short paper describing his solution [5]. The fact that Euler could solve the problem is not that interesting to us, but rather the intermediate step that he took in the process: Euler had the foresight to simplify his problem by considering the problem as a *graph* consisting of *links* and *nodes*. The nodes in this case are the four pieces of land and the links are the seven bridges connecting the nodes. Once this is done it is easy to proof generally that if a node has an odd number of links then it must be either a starting point or an end point of the journey. This means that there cannot be more than two nodes with odd number of links and in the case of the bridges there were four i.e. all of the nodes had odd number of links. Thus the path cannot exist even if we spent our life searching for it. This is important to notice: It does not depend on our ability to find such a path; rather it is a *property of the graph*. Finally the people of Königsberg agreed with Euler and later on they built yet another bridge, solving the problem.

There were many consequences to this intermediate step Euler took in his proof. First of all it launched an avalanche of studies and contributions made by mathematical giants such as Cauchy, Hamilton, Cayley, Kirchoff and Pólya. On the other hand it started the *thinking of networks*. The fact that there are many types of networks (or graphs at that time) and that they can have properties hidden in their construction that enable or restrict the ability to do different things with them was acknowledged. Also, by making even very small changes to these properties we can drastically change the behavior of the network and perhaps open hidden doors that enable new possibilities to emerge as was the case in Königsberg; just by adding a single link to the graph the whole problem was solved.

After Euler's work the main goal of graph theory was to discover and catalogue various graphs and their properties. For example labyrinth problems, first solved in 1873, were very famous at that time along with problems like finding a sequence of moves with a knight on a chess board such that each square is visited only once and that the knight returns to its starting point. Also things like the lattice formed by atoms in a crystal or the hexagonal lattice made by bees in a beehive were studied.

6.2.2 Random networks

It took two centuries after the advent of graph theory before scientists started moving from studying various properties of different graphs to asking how networks form. What are the laws governing their appearance, structure and changes in them? The first answers to these questions were given by two Hungarian mathematicians Paul Erdos and Alfréd Rényi when in 1959 they published their paper about random networks [9].

The idea behind random networks was that when creating a network the links between nodes were added completely randomly. Once they started adding random links one by one they also noticed that when the average amount of links per node approached one, something special happened; a single network emerged. Physicists would perhaps call this phenomenon a phase transition, similar to the moment in which water freezes. Sociologists would say that your subjects had just formed a community, but all would agree that something special happened: After placing critical amount of links, the network drastically changed. The properties of random networks were further studied and one of Erdos's students, Béla Bollobás, proofed in 1982 that the histogram of links per node in a random network follows a Poisson distribution [6].

Poisson distribution tells us that the majority of the nodes have the same number of links as the average node has (peak value of the distribution) and that it is extremely rare that there exists a node in the graph that would deviate significantly from this "prototype node". When translated for example to society, we end up with very democratic world in which everyone has on the average the same number of friends or other social links. It tells that most companies trade with roughly the same number of companies, most neurons connect to roughly the same number of neurons, most Websites are visited by roughly the same number of people and so on.

But is this the way that nature truly behaves? Of course it is not. Although Erdos and Rényi understood that there is very diverse spectrum of different kinds of networks in the universe they deliberately disregarded this diversity and proposed the simplest possible way nature could follow: Randomness. They never tried to solve all the network-problems at once but to propose a general approach to certain problems by introducing randomness to their model. This observation can also be noticed from their 1959 seminal paper: *"the evolution of graphs may be considered as a rather simplified model of the evolution of certain communication nets (railway, road or electric network systems, etc.)"* [9]. Still their work inspired the research of the theory behind complex networks and since for a long time there was no better approach available, often complex networks were considered as fundamentally random.

6.3 Degrees of separation

One interesting aspect in networks that has also intrigued scientists over the decades is their smallness; even if the network at hand is very large, still often it seems to be so interconnected that the average distance between nodes is surprisingly small. One of the most important concepts in this area is the so called *"six degrees of separation"* [13]. The concept was born from the studies of society made by Frigyes Karinthy. In his book "Láncszemek" (or "Chains" in english) he proposes that everybody is at most five handshakes away from each other. He presents few interesting examples in which he can connect himself to any Nobelist or to any worker in Henry Ford's car factory.

Although the *"six degrees of separation"* did not draw too much attention in Karinthy's time it was rediscovered almost three decades later, in 1967, by Stanley Milgram, a Harvard professor who turned the concept into famous study on our interconnectivity [21]. He modified Karinthy's idea so that he could estimate the "distance" between any two people in United States. In his experiment he sent the following letter along with other information to 160 randomly chosen people living in Wichita and Omaha.

HOW TO TAKE PART IN THIS STUDY

1. ADD YOUR NAME TO THE ROSTER AT THE BOTTOM OF THIS SHEET, so that the next person who receives this letter will know who it came from.
2. DETACH ONE POSTCARD. FILL IT OUT AND RETURN TO HARVARD UNIVERSITY. No stamp is needed. The postcard is very important. It allows us to keep track of the progress of the folder as it moves toward the target person.

3. IF YOU KNOW THE TARGET PERSON ON A PERSONAL BASIS, MAIL THIS FOLDER DIRECTLY TO HIM (HER). Do this only if you have previously met the target person and know each other on a first name basis.

4. IF YOU DO NOT KNOW THE TARGET PERSON ON A PERSONAL BASIS, DO NOT TRY TO CONTACT HIM DIRECTLY. INSTEAD, MAIL THIS FOLDER (POSTCARDS AND ALL) TO A PERSONAL ACQUAINTANCE WHO IS MORE LIKELY THAN YOU TO KNOW THE TARGET PERSON. You may send the folder to a friend, relative or acquaintance, but it must be someone you know on a first name basis.

After sending the letters, Milgram wondered if any of those letters would find their path to the receiver and if they did how many links would it take? He consulted also other well-educated people with his question and their best estimate was close to 100. To their great surprise, first letter arrived within a few days, passing through only two intermediate links! This turned out to be the shortest path ever recorded. Eventually 42 of the 160 letters made it back and the median number of intermediate persons turned out to be 5.5. Quite a small number compared to what he and his well-educated friends came up with. Although the results of Stanley Milgram's experiment have been questioned (see [18] and [17]), they provide a clear evidence that the distance between any two people in a "social web" is relatively small.

Degrees of separation have also been studied in many other networks: For example the species in food web are separated by two degrees of separation ([28], [22]), molecules in a cell are connected through three chemical reactions ([15], [14], [26]), scientists through four co-authorships ([23]) and neurons in the brain of the Caenorhabditis elegans worm through 14 synapses ([27]) an so on. However, one could argue that the experiments performed with social networks and with so small number of data points, could be very far from the truth. This suspicion can today be addressed with enormous databases that are kept on various things. For example social networks among actors and actresses have been studied utilizing the Internet Movie Data Base (*www.imdb.com*), which contains virtually every actor or actress in the world that has appeared in a movie. Also the modern computer networks that are connected through routers, the largest being the Internet, are a good source of well documented information for these studies. In fact, the degree of separation for the Internet is 10. The largest degree of separation found today is 19 and it was identified for the highly interconnected network of billion plus documents that are connected to each other through hyperlinks in the World Wide Web (WWW) [2].

The problem can also be approached completely analytically by using the theory of random networks. It can be shown that generally the degree of separation d for random networks is the following: $d = \frac{\log N}{\log k}$, where $N =$ size of the net and $k =$ average number of links per node. As can be seen from the logarithmic behavior of d, the degree of separation grows very slowly even if the size of the network becomes large. This tells us in clear terms of mathematics that when it comes to interconnected networks; *“we live in a small world”*.

6.4 Hubs and connectors

A staff writer at the *New Yorker* magazine called Malcolm Gladwell wrote a book named *The Tipping Point* where he introduces a simple test: The task is to go through a list of 248 surnames that he had compiled from Manhattan phone book and count the number of people from your own circle of friends and acquaintances that have their surname appearing on the list, counting also multiple occurrences [11]. The purpose of the test was to measure the social connectedness of different people. Gladwell tested about four hundred people including several different types and also very homogenous groups of people (similar age, education and income). The average was changing according to education, age and social status of different groups and was somewhere between 20 and 40. This was not so important, however, and could be easily explained but the interesting thing in these experiments was that the variation in the results was very large and independent of the group or the homogeneity of the people in the group. The lowest score was typically between 2 and 16 and the highest 95 and 118. Gladwell found that in every test there are a small number of people that have the special ability to make friends and acquaintances more easily than others. He called these people *connectors* but in network terms they can be called *hubs*. Same kind of phenomena has been studied in many different networks (e.g. WWW, Internet, social nets) [16], [10].

Yet, the random universe discussed above does not support connectors. If the networks were random then the existence of the hubs would be practically forbidden due to their increasingly small probability. If the WWW were a random network, the probability of there being a page with five hundred incoming links would be 10^{-99}. Yet the latest Web survey, covering roughly 20% of the full Web, found four hundred such documents. The document with most incoming links had over two million links. The chance of finding such a document from a random network is smaller than locating an individual atom in the universe. Thus, new laws for describing networks had to be found.

6.5 The Scale-Free Networks

When researchers were studying the WWW it was found that the number of links between HTML Hyper Text Markup Language) pages did not follow a uniform distribution. From real data collected by robots crawling through the WWW it was found that when the data was plotted on a log-log scale it was a straight line. This was a strong indication that the distribution follows a *power law [7]*. This implies that the number of WebPages with exactly k links, denoted by $N(k)$, follows $N(k) \sim k^{-\gamma}$, where the parameter γ is the degree exponent. For WWW the γ was recorded to be 2.1 for incoming links and 2.5 for outgoing links [2].

Same kind of behavior was found in many other networks also. For example by utilizing the vast database of movies (The Internet Movie Database, *www.imdb.com*) it was found that the number of actors that had links to exactly k other actors decays following a power law [10]. Another example is the cell where the number of molecules interacting with exactly k other molecules follows also a power law [26].

6.5.1 The 80/20 Rule

One interesting implication of the existence of the power laws in certain systems has been around for a long time — from the beginning of the 20^{th} century — and is known as the 80/20 rule. The rule was invented by an influential Italian economist called Vilfredo Pareto. Pareto was an avid gardener and he happened to notice that 80 percent of his peas were produced by only 20 percent of the peapods. He made also other similar observations, for example he noticed that 80 percent of Italy's land was owned by only 20 percent of the population and so on. More recently Pareto's Law or Principle got the name 80/20 rule and turned into the Murphy's Law of management: 80 percent of profits are produced by only 20 percent of employees, 80 percent of customer service problems are produced by 20 percent of consumers, 80 percent of decisions are made during 20 percent of the meeting time, 80 percent of crime is committed by 20 percent of criminals, and so on [19].

The 80/20 rule is a clear implication of the existence of the power laws in certain systems: It says that there are a smaller number of strong influencers (the hubs) in the system and the others are not so important.

6.5.2 Random and scale-free networks

Of course not all networks follow a power law distribution. There are many different networks in the universe that have nothing to do with power laws. But there exists also many real networks that do and they are called *scale-free networks*. The name comes from the fact that in networks that follow a power law there is no intrinsic scale in them. There is no single node that we could pick up and say that this is the prototype-node of this network, like we can in a network that follows a Bell curve.

A good way to visualize this is to consider the differences between a random network and one described by a power-law degree distribution. Let's compare a U.S. roadmap with an airline routing map. In the roadmap cities are the nodes and the highways connecting them are the links. As one can imagine the roadmap is fairly uniform network: Each major city has at least one link to the highway system and on the other hand there are no cities that would have hundreds of links. Another graph is the airline routing map where the situation is completely different. In this map airports (nodes) are connected to other airports by direct flights between them (links). In this case it is clear that there exists a few major airports in the country that have a high number of flights and a large number of smaller airports that have only few flights. When the two networks are compared it is clear that they have very different properties and this is why they also look very different. The roadmap follows a Bell curve telling that vast majority of the nodes has the same number of links and that it is extremely rare to find a node that would differ greatly from the average, which is located at the peak value of the degree distribution. On the other hand the airline map follows a power law distribution and there is no single node that could be selected as a prototype of the network. Rather there exist few major influencers (hubs) that take care of most of the traffic and shape the network to be scale-free. As can be seen, scale-free networks are one special case of networks and they define the topology of the network to such that there is no prototype-node but the hubs in the net define the behavior [3].

6.5.3 Robustness vs. vulnerability

Let us consider the two types of networks presented above, and think them as communication networks where each node is a communication point and the messages can be sent via links between nodes. Which would be more reliable?

As usual, the answer is not unambiguous: If we consider the robustness against random errors (equipment failures for example) then it has been studied that the scale-free networks are extremely error tolerant. It has been found that typically nearly 80% of the nodes can be randomly removed until the network is crippled. On the other hand random network is fairly easily halted. This difference in behavior is because of the connecting force of the hubs in the scale-free network. Since the probability of error occurring in a node is exactly the same for every node in the network, the scale-free network can afford to lose a large amount of the “less important” nodes without significant loss in performance.

There is a tradeoff however, as the increasing number of hubs in the network increases its stability against random errors at the same time its tolerance against targeted attacks is decreased. This is clear because of the important role of the hubs in the network and is exactly how the *MafiaBoy* accomplished in his attack (as described in the beginning). The attack was targeted at the most important hubs and thus the damage was enormous. This tradeoff is very important to notice and should be considered carefully in network design. Obviously the selection of the network topology should be based on the boundary conditions (intended use, security aspects and so on) [1].

6.6 Viruses and fads

Another interesting and much studied property of most networks is their ability to rapidly spread information. In most cases it is a good and hoped phenomenon, but there are cases where this kind of behavior can lead to unwanted results (i.e. computer viruses, spreading of different diseases like AIDS or SARS more recently). There are, however, many unanswered questions, like: Why do some make it and others just “crash and burn”? How could this behavior be predicted in a network? Could it be controlled reliably?

Again, it is important to understand the properties of the underlying network when studying these things. Aiming to explain the disappearance of some fads and viruses and the spread of others, social scientists and epidemiologists have developed a tool called the *threshold model.* The model is based on the fact that all people differ in their willingness to adopt a new idea. But in general, with sufficient positive evidence everyone can be convinced. Only the level of positive evidence differs. Thus, the model gives every individual (or generally every node) an individual threshold value indicating the likelihood that he or she will adopt a given innovation. Another parameter called *critical threshold* can be calculated utilizing the properties of the network and thus a *spreading rate* for a given innovation can be estimated [12].

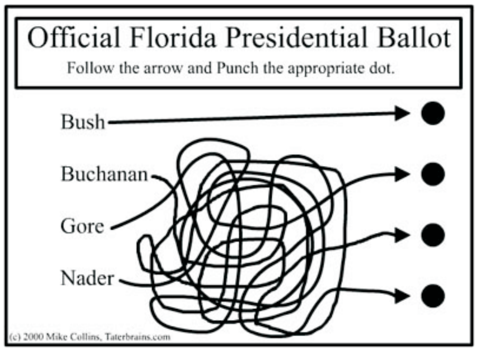

Figure 6.2: Mike Collins's cartoon

One interesting example of the rapid spreading rate was the frustration of a twenty-six-year old municipal water board engineer to the confusion in the presidential elections in Florida in 2000. The man was called Mike Collins and to make fun of the situation he sketched a picture like the one shown in Figure 6.2 and sent it to thirty friends through e-mail. The next day was his birthday and the day his sister gave birth to a daughter so he was out all day. When he came home, a huge surprise waited: 17 000 new hits on his Webpage and several hundred e-mails. While he was away his nipping cartoon had circled the globe and newspapers and Websites from United States to Japan were bombarding him with requests for permission to publish. In a few hours he went from unknown “Mike” to an instant celebrity, with girls hitting on him and parents wanting to fix him up with their daughters. On the other hand when some commercial companies try to do similar things with their products, they often fail even if they spend huge amounts of money while trying [20].

It seems that the threshold model is quite powerful tool for estimating spreading of different things in networks: Epidemiologists use it to model the probability that a new infection will turn into an epidemic; Marketing people estimate how a new product will do in the markets; sociologists use it to explain the spread of birth control practices among women; Political science exploits it to explain the life cycle of parties and movements or to model the likelihood that peaceful demonstrations turn into riots

6.7 The Map of Life

"Today we are learning the language in which God created life", said President Bill Clinton on June 25, 2000, at the White House ceremony announcing the decoding of the 3 billion chemical "letters" of human genome. Was he right? Now that we have the full DNA of a human being figured out, don't we have all the information needed to cure different diseases and estimate medical facts? The answer is: No! And the reason should be clear by now. As explained earlier when talking about reductionism, we run again into the *hard wall of complexity*. We only have the pieces but not the blueprint. To cure most illnesses, we need to understand living systems in their integrity. We need to understand how and when different genes work together, how messages travel within the cell, which reactions are taking place or not in this complex cellular network.

However, studies have shown that there is scale–free–network–behavior present also in metabolic networks: For example, a study conducted on 43 different organisms on how many reactions each molecule participates in showed that all the nets were scale–free with three degrees of separation [15]. There are also many other examples showing that networks are present in living organisms [25]. So the question is; if we improve our *"network-thinking"* abilities can we also improve our understanding of living systems? Many researchers seem to think that we can.

6.8 Conclusions

Networks are present almost everywhere, all we need is an eye for them. Perhaps if we shift our way of thinking towards network-world we can open up new doors for science. One interesting area of research would be the dynamical models of the changes taking place in different networks. After all, most of the real networks are under continuous change and the networks describing them are only the skeletons of complexity. *Identifying the underlying structures and their properties gives us a map for our journey through complex worlds!*

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