Session 9

. . . Towards New Order

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In this chapter we are moving from chaos *towards new order i.e.* complexity*. The complexity lies at the* edge of chaos *(EOC), which is the border area between static and chaotic behavior. Other subjects in this chapter such as* self-organized criticality *(SOC),* highly optimized tolerance *(HOT) and measures of complexity are closely related to EOC.*

9.1 Introduction to complexity

It is obvious that chaos theory existed long before anyone spoke about complexity. But there was a field called "Catastrophe theory" even before chaos. Catastrophe theory studies and classifies phenomena characterized by sudden shifts in behavior arising from small changes in circumstances. Originated by the French mathematician Rene Thom in the 1960s, catastrophe theory is a special branch of dynamical systems theory [1]. Anyway, as James P. Sethna from Cornell University, Ithaca says: ". . . the big fashion was topological defects. Everybody was . . . finding exotic systems to write papers about. It was, in the end, a reasonable thing to do. The next fashion, catastrophe theory, never became important for anything" [2].

The successor of catastrophe theory, chaos theory has been described in the previous chapter. In this chapter the emphasis is on the latest research area of this field, i.e. *complexity* and *complex systems*. There has been a lot of discussion whether complexity is any different from chaos. Is there really something new or are we talking about the same subject with a new name? Langton's ("father" of EOC) famous egg diagram in Fig. 9.1 gives some insight to this matter. The behavior of systems has been divided into four different classes in the egg. These are "Fixed", "Periodic", "Complex" and "Chaotic". It is justified to say that complex behavior is different from chaotic behavior. Thus complexity is not chaos and there is something new in this field. But yet the things studied in the field of complexity are somehow familiar from the past.

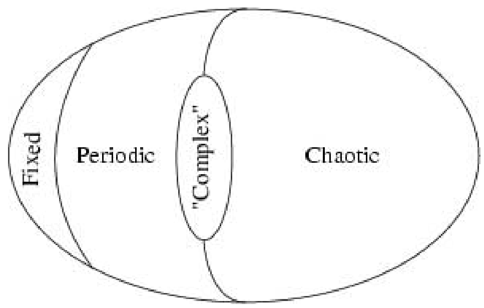

Figure 9.1: Langton's famous egg diagram [3]

The advances in the scientific study of chaos have been important motivators and roots of the modern study of complex systems. Chaos deals with deterministic systems whose trajectories diverge exponentially over time and the models of chaos generally describe the dynamics of one or few variables that are real. Using these models some characteristic behaviors of their dynamics can be found. Complex systems do not necessarily have these behaviors. Complex systems have many degrees of freedom, many elements that are partially but not completely independent. Complex behavior can be seen as "high dimensional chaos". Chaos is concerned with a few parameters and the dynamics of their values, while the study of complex systems is concerned with both the structure and the dynamics of systems and their interaction with their environment [4].

Complexity can be defined in many ways. We have different definitions depending on the field of science. For example, in the field of information technology complexity is defined to be either the minimal length of a description of a system (in information units) or the minimal time it takes to create the system [4]. On the other hand, complexity necessarily depends on the language that is used to model the system. The original Latin word complexus signifies "entwined" or "twisted together". This may be interpreted in the following way: in order to have a complex you need two or more components, which are joined in such a way that it is difficult to separate them [5].

Another important term in the field of complexity is *emergence*, which is [4]

1. What parts of a system do together that they would not do by themselves (collective behavior). How behavior at a larger scale of the system arises from the detailed structure, behavior and relationships on a finer scale.
2. What a system does by virtue of its relationship to its environment that it would not do by itself.
3. Act or process of becoming an emergent system.

Emergence refers to all the properties that we assign to a system that are really properties of the *relationship* between a system and its environment.

9.2 Self-organized criticality

In this section, self-organized criticality (SOC) is described. SOC has its roots in fractals. So far we have discussed fractals only as a geometrical phenomena, but in SOC case the dynamics is important. The SOC is often demonstrated by using the sand pile model, which is also discussed here.

9.2.1 Dynamical origin of fractals

Many objects in nature are best described geometrically as fractals with self-similar features on all length scales. In nature, there are for example mountain landscapes that have peaks of all sizes, from kilometers down to millimeters, river networks that have streams of all sizes, and earthquakes, which occur on structures of faults ranging from thousands of kilometers to centimeters. Fractals are scale-free so you can't determine the size of a picture of a part of a fractal without a yardstick. The interesting question is then how nature produces fractals?

The origin of the fractals is a dynamical, not a geometrical problem. Geometrical characterization of fractals has been widely examined but it would be more interesting to gain understanding of their dynamical origin. Consider for example earthquakes. They last for a few seconds, but the fault formations in the crust of the earth are built up during some millions of years. The crust seems static if the observation period is only a human lifetime. The laws of physics are local, but fractals are organized over large distances. Large equilibrium systems operating near their ground state tend to be only locally correlated. Only at a critical point where continuous phase transition takes place are those systems fractal [6].

9.2.2 SOC

Per Bak, Chao Tang, and Kurt Wiesenfeld introduced the concept of self-organized criticality in 1987. SOC refers to tendency of large dissipative systems to drive themselves to a critical state with a wide range of length and time scales. As an example, consider a damped spring as in Fig. 9.2. A mass m is attached to the spring, which is fastened into the wall on the left. The spring constant is k and the damping coefficient is B. The distance from the equilibrium point is $x(t)$.

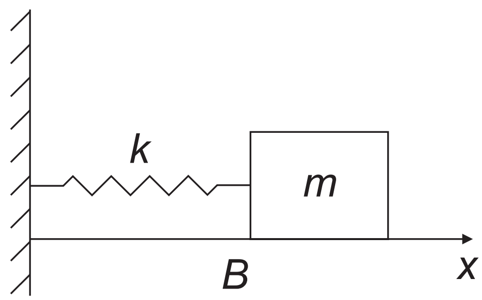

Figure 9.2: A damped spring.

From the first principles it is easy to model the spring system traditionally.

$$m\ddot{x}(t) + B\dot{x}(t) + kx(t) = 0 \qquad \Rightarrow \ddot{x}(t) = -\frac{1}{m} \cdot [B\dot{x}(t) + kx(t)] \tag{9.1}$$

It is also possible to simulate the system. To do that, the following initial conditions and constants have been defined.

$$\begin{cases} k = 0.1\frac{N}{m} \\ B = 0.08\frac{kg}{s} \\ m = 1kg \\ \dot{x}(0) = 1\frac{m}{s} \end{cases} \tag{9.2}$$

The *Simulink* model of the spring is presented in Fig. 9.3.

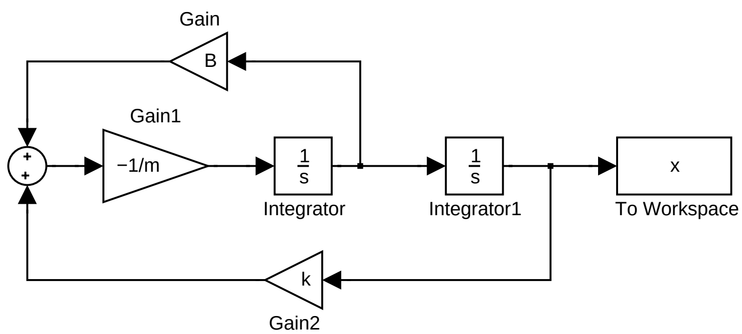

Figure 9.3: *Simulink* model of the spring.

After the simulation (200 seconds) the response $x(t)$ is plotted in Fig. 9.4. Figure 9.5 represents the last 40 seconds of the simulation (on a finer scale!).

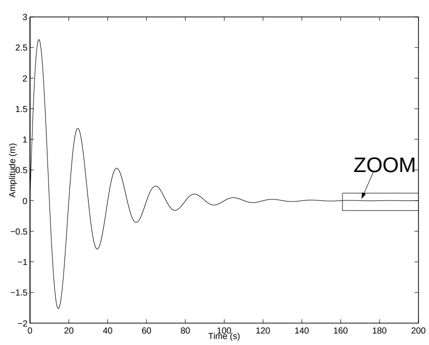

Figure 9.4: The response $x(t)$.

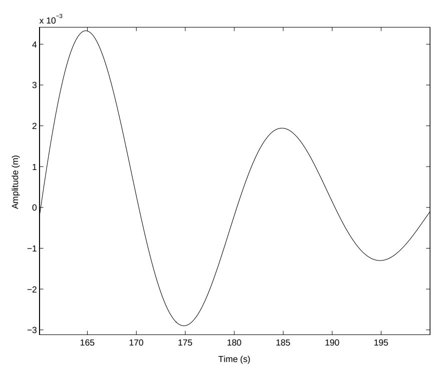

Figure 9.5: A closer look at the damping.

The scale is very different in the two plots. As we can see from the simulation results, the oscillatory behavior of the spring with a decreasing amplitude theoretically continues forever. In real world, the motion would stop because of the imperfections such as dust. Once the amplitude gets small enough, the emotion suddenly stops. This generally occurs when the velocity is smallest i.e. at the “top” or at the “bottom” of an oscillation. This is not the state of smallest energy! In a sense, the system is most likely to settle near a "minimally stable" state. In that state the system still has some potential energy. If the spring stops at the top of the oscillation, some energy remains in the spring. Then the system becomes very sensitive for small perturbations because a little "push" can free the energy and the spring may start oscillating again.

This kind of behavior can be detected as well when analyzing pendulums. Consider a system with coupled, multiple pendulums. If the system is in steady-state with all the pendulums in a minimally stable state, a small perturbation can avalanche the whole system. Small disturbances could grow and propagate through the system with little resistance despite the damping and other impediments. Since energy is dissipated through the process, the energy must be replenished for avalanches to continue. If self-organized criticality is considered, the interest is on the systems where energy is constantly supplied and eventually dissipated in the form of avalanches [6].

The dynamics in the SOC state is intermittent with periods of inactivity separated by well-defined bursts of activity or avalanches. The critical state is an attractor for the dynamics. The SOC idea provides a unifying concept for large-scale behavior in systems with many degrees of freedom. SOC complements the concept of chaos wherein simple systems with a small number of degrees of freedom can display quite complex behavior. Large avalanches occur rather often and there is no exponential decay of avalanche sizes, which would result in a characteristic avalanche size, and there is a variety of power laws without cutoffs in various properties of the system [7].

The paradigm model for this type of behavior is the celebrated sand pile cellular automaton also known as the Bak-Tang-Wiesenfeld (BTW) model.

9.2.3 Sand pile model

Adding sand slowly to a flat sand pile will result only in some local rearrangement of particles. The individual grains, or degrees of freedom, do not interact over large distances. Continuing the process will result in the slope increasing to a critical value where an additional grain of sand gives rise to avalanches of any size, from a single grain falling up to the full size of the sand pile. The pile can no longer be described in terms of local degrees of freedom, but only a holistic description in terms of one sand pile will do. The distribution of avalanches follows a power law.

If the slope of the pile were too steep, one would obtain a large avalanche and a collapse to a flatter and more stable configuration. If the slope were too shallow, the new sand would just accumulate to make the pile steeper. If the process is modified, for instance by using wet sand instead of dry sand, the pile will modify its slope during a transient period and return to a new critical state. Consider for example snow screens. If they were built to prevent avalanches, the snow pile would again respond by locally building up to steeper states, and large avalanches would resume.

The sand pile model can be simulated and there are some Java-applets available on the Internet (e.g. [8]). Typically, the simulation model is such that there is a 2-D lattice (cellular automaton) with N sites. Integer values z_i represent the local sand pile height at each site i. If the height exceeds some critical height z_{cr} (let's say three), then one grain is transferred from the unstable site to each of the four neighboring site (this is called a toppling). A toppling may initiate a chain reaction, where the total number of topplings is a measure of the size of an avalanche. To explore the SOC of the sand pile model, one can randomly add sand onto the pile and have the system relax. The result is unpredictable and one can only simulate the resulting avalanche to see the outcome. After adding a large amount of sand, the configuration seems random, but some subtle correlations exist (e.g. never do two black cells lie adjacent to each other, nor does any site have four black neighbors). Avalanche is triggered if a small amount of sand is added to a site near the center. The size of an avalanche is calculated and the simulation is repeated. After the simulations, it is possible to analyze the distribution of the avalanche sizes. The distribution of avalanche size follows a power law

$$P(s) = s^{1-\tau}, \tau \cong 2.1 \tag{9.3}$$

where s is the size of an avalanche and $P(s)$ is the probability of having an avalanche of size s.

Because of the power law, the initial state was actually remarkably correlated although it was thought not to be that. For random distribution of z's (pile heights), one would expect an avalanche to be either sub-critical (small avalanche) or supercritical (exploding avalanche with collapse of the entire system). Power law indicates that the reaction is precisely critical, i.e. the probability that the activity at some site branches into more than one active site, is balanced by the probability that the activity dies [6].

9.3 Complex behavior and measures

In this section two approaches to the *edge of chaos* phenomenon are discussed as well as the measures of complexity. Phase transitions and their relation to Wolfram's four classes (discussed in the earlier chapters) are also presented.

9.3.1 Edge of chaos — Langton's approach

The edge of chaos is the interesting area, where complex rather than chaotic or static behavior arises. Backed by Kauffman's work on co-evolution, Wolfram's cellular automata studies, and Bak's investigations of self-organized criticality, Langton has proposed (in 1990) the general thesis that complex systems emerge and maintain on the edge of chaos, the narrow domain between frozen constancy and chaotic turbulence. The edge of chaos idea is another step towards an elusive general definition of complexity [9].

In general, some cellular automata are boring since all cells either die after few generations or they quickly settle into simple repeating patterns. You could call these "highly ordered" cellular automata because their behavior is predictable. Other cellular automata are boring because their behavior seems to be totally random. You could call these "chaotic" cellular automata because their behavior is totally unpredictable. On the other hand, some cellular automata show interesting (complex, lifelike) behavior, which arises near the border of between chaos and order. If these cellular automata were more ordered, they would be predictable, and if they were less ordered, they would be chaotic. This boundary is called the edge of chaos.

Christopher Langton introduced a 1-D cellular automaton, whose cells have two states. The cells are either "alive" or "dead". If a cell and its neighbors are dead, they will remain dead in the next generation. Langton defined a simple number that can be used to help predict whether a given cellular automaton will fall in the ordered realm, in the chaotic realm, or near the boundary. The number *lambda* ($0 < \lambda < 1$) can be computed from the rule space of the cellular automaton. The value is simply the fraction of rules in which the new state of the cell is living. The number of rules R of a cellular automaton is determined by $R = K^N$, where K is the number of possible states and N is the number of neighbors. If $\lambda = 0$, the cells will die immediately and if $\lambda = 1$, any cell with a living neighbor will live forever. Values of λ close to zero give cellular automata in the ordered realm, and values near one give cellular automata in the chaotic realm. The edge of chaos is somewhere in between.

Value of λ does not simply represent the edge of chaos. It is more complicated. You could start with $\lambda = 0$ (death) and add randomly rules that lead to life instead of death ($\Rightarrow \lambda > 0$). You would get a sequence of cellular automata with values of λ increasing from zero to one. In the beginning, the cellular automata would be highly ordered and in the end they would be chaotic. Somewhere in between, at some critical value of λ, there would be a transition from order to chaos. It is near this transition that the most interesting cellular automata tend to be found, the ones that have the most complex behavior. The critical value of λ is not a universal constant [10].

9.3.2 Edge of chaos — another approach

We can approach the edge of chaos from another point of view. This view is based on measuring so called *perturbation strength*. Let us consider some examples. The first one is domino blocks in a row. The blocks are first in a stable state, standing still. The initial state is in a way minimally stable, because a small perturbation can avalanche through the whole system. Once the first block is nudged, an avalanche is started. The system will become stable again once all blocks are lying down. The nudge is called *perturbation* and the duration of the avalanche is called *transient*. The strength of the perturbation can be measured in terms of the effect it had i.e. the length of time the disturbance lasted (or the *transient length*) plus the permanent change that resulted (none in the domino case).

Other examples of perturbation strength are buildings in earthquakes, and air molecules. For buildings, we require short transient length and return to the initial state (buildings are almost static). Air molecules on the other hand collide with each other continually, never settling down and never returning to exactly the same state (molecules are chaotic). For air molecules the transient length is infinite, whereas for our best buildings it would be zero. How about in the middle?

Consider yet another example, a room full of people. A sentence spoken may be ignored (zero transient), may start a chain of responses which die out and are forgotten by everyone (a short transient) or may be so interesting that the participants will repeat it later to friends who will pass it on to other people until it changes the world completely (an almost infinite transient – e.g. old religions).

Systems with zero transient length are static and systems with infinite transient length are chaotic. The instability with order as described in the examples is called the edge of chaos, a system midway between stable and chaotic domains. EOC is characterized by the potential to develop structure over many different scales and is often found feature in those complex systems whose parts have some freedom to behave independently. The three responses in the room example could occur simultaneously, by affecting various group members differently. The idea of transients is not restricted in any way and it applies to different type of systems, e.g. social, inorganic, politic, and psychological... Hence we have a possibility to measure totally different type of systems with the same measure. It seems that we have a quantifiable concept that can apply to any kind of system. This is the essence of the complex systems approach, ideas that are universally applicable [11].

9.3.3 Complexity measures

In previous section transient length was presented and it was concluded to be a universal measure for complex systems. Another measure is *correlation distance*. Correlation in general, is a measure of how closely a certain state matches a neighboring state. The correlation can vary from 1 (identical) to -1 (opposite). For a solid we expect to have a high correlation between adjacent areas, but the correlation is also constant with distance. For gases correlation should be zero, since there is no order within the gas because each molecule behaves independently. Again the distance is not significant, zero should be found at all scales.

Each patch of gas or solid is statistically the same as the next. For this reason an alternative definition of transient length is often used for chaotic situations i.e. the number of cycles before statistical convergence has returned. When we can no longer tell anything unusual has happened, the system has returned to the steady-state or equilibrium. Instant chaos would then be said to have a transient length of zero, the same as a static state, since no change is ever detectable.

For complex systems one should expect to find neither maximum correlation (nothing is happening) nor zero (too much happening), but correlations that vary with time and average around midway. One would also expect to find strong short-range correlations (local order) and weak long range ones. Thus we have two measures of complexity: correlations varying with distance and long non-statistical transients [11].

9.3.4 Phase transitions

Phase transition studies came about from the work begun by John von Neumann and carried on by Steven Wolfram in their research of cellular automata. Consider what happens if we heat and cool systems: at high temperatures systems are in gaseous state (chaotic) and at low temperatures systems are in solid state (static). At some point between high and low temperatures the system changes its state between the two i.e. it makes a phase transition [11].

There are two kinds of phase transitions: first order and second order. First order we are familiar with when ice melts to water. Molecules are forced by a rise in temperature to choose between order and chaos right at $0\,°C$. This is a deterministic choice. Second order phase transitions combine chaos and order. There is a balance of ordered structures that fill up the phase space. The liquid state is where complex behavior can arise [12].

Consider once again the "egg diagram" in Fig. 9.1. It shows the schematic drawing of cellular automaton rule space indicating relative location of periodic, chaotic, and complex transition regimes. Crossing over the lines (in the egg) produces a discrete jump between behaviors (first order phase transitions). It is also possible that the transition regime acts as a smooth transition between periodic and chaotic activity (like EOC experiments with λ). This smooth change in dynamical behavior (smooth transition) is primarily second-order, also called a critical transition [3].

In his research, Wolfram has divided cellular automata into four classes based on their behavior. Different cellular automata seem to settle down to classes which are: constant field (Class I), isolated periodic structures (Class II), uniformly chaotic fields (Class III) and isolated structures showing complicated internal behavior (Class IV). There is a relationship between Wolfram's four classes and the underlying phase transition structure in cellular automata's rule space [13]. Figure 9.6 represents a schematic drawing of the relationship.

Phase transition feature allows us to control complexity by external forces. Heating or perturbing the system drives it towards chaotic behavior, and cooling or isolating the system drives it towards static behavior. This is seen clearly in relation to brain temperature. Low temperature means static behavior (hypothermia), medium temperature normal, organized behavior and high temperatures chaotic behavior (fever).

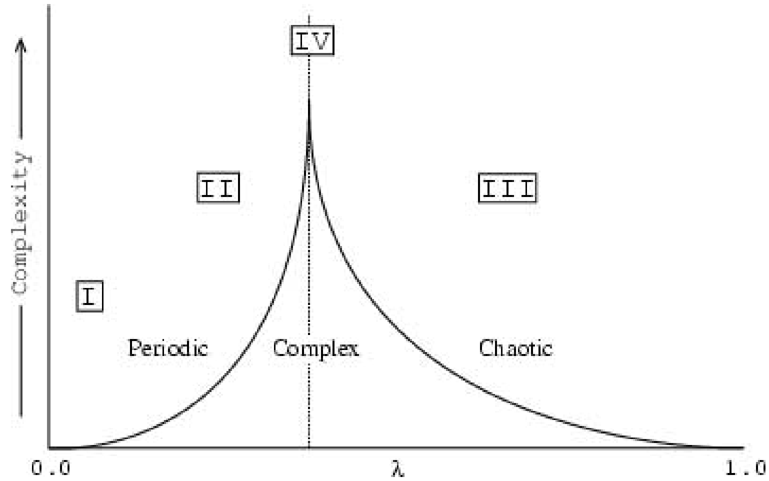

Figure 9.6: A schematic drawing of the cellular automata's rule space showing the relationship between Wolfram's classes and the underlying phase transition structure. [3]

9.4 Highly optimized tolerance

John Doyle's contribution to the complex systems field is "highly optimized tolerance". It is a mechanism that relates evolving structure to power laws in interconnected systems. HOT systems arise, e.g. in biology and engineering where design and evolution create complex systems sharing common features, such as high efficiency, performance, robustness to designed-for uncertainties, hypersensitivity to design flaws and unanticipated perturbations, non-generic, specialized, structured configurations, and power laws. Through design and evolution, HOT systems achieve rare structured states that are robust to perturbations they were designed to handle, but yet fragile to unexpected perturbations and design flaws. As an example of this, consider communication and transportation systems. These systems are regularly modified to maintain high density, reliable throughput for increasing levels of user demand. As the sophistication of the systems is increased, engineers encounter a series of tradeoffs between greater productivity and the possibility of catastrophic failure. Such robustness tradeoffs are central properties of the complex systems that arise in biology and engineering [14].

Robustness tradeoffs also distinguish HOT states from the generic ensembles typically studied in statistical physics under the scenarios of the edge of chaos and self-organized criticality. Complex systems are driven by design or evolution to high-performance states that are also tolerant to uncertainty in the environment and components. This leads to specialized, modular, hierarchical structures, often with enormous "hidden" complexity with new sensitivities to unknown or neglected perturbations and design flaws.

An example of HOT system design is given here to enlighten the idea. Often the HOT examples deal with the forest fire model, thus it is described here as well. Consider a square lattice where forest is planted. The forest density on the lattice can be something between zero and one. Zero means no forest at all and one means that the lattice is full of trees. Assume that a "spark" hits the lattice at a single site. A spark that hits an empty site does nothing, but a spark that hits a cluster of trees burns the whole cluster (see Fig. 9.7) [14], [15].

The yield of the forest after the hit of the spark is determined to be the forest density minus the loss (burned forest). In ideal case (no sparks) the yield of the forest grows linearly as a function of density. But if there are sparks, the linearity holds only for values lower than approximately 0.55. If the density of the forest exceeds a critical point ($\rho = 0.5927$), the yield decreases rapidly if forest density is yet increased. This is, of course, because the forest becomes too dense and the tree clusters are almost fully connected (the sizes of tree clusters increase). Figure 9.8 represents the yield as a function of forest density on an infinite lattice (lattice size $N \to \infty$) [15].

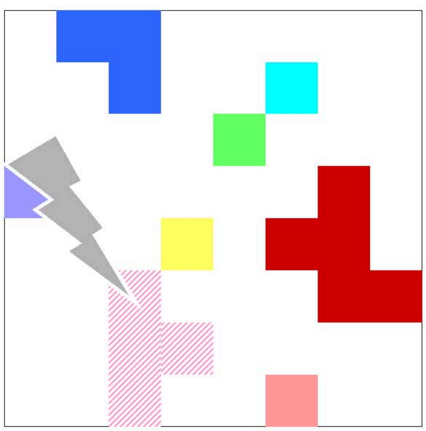

Figure 9.7: A square lattice and a spark hitting a cluster of trees. [15]

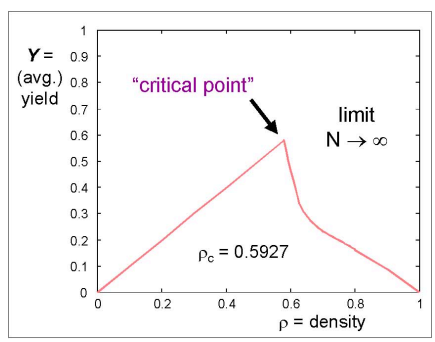

Figure 9.8: Forest yield as a function of density. [15]

According to Fig. 9.8 the sparks do not matter if the density of the forest is less than the critical density (0.5927). If the forest density is higher, the whole forest might get burned because of one spark hit anywhere in the forest.

If HOT is compared with SOC or EOC, we clearly see the difference. SOC and EOC assume that the interesting phenomena are at criticality. In HOT state systems actually work over the critical point, where their performance is optimized. In the forest model example this means that forest can be planted denser than the critical density. In that way also the yield of the forest can be increased, and the system is said to run in a HOT state. The idea is to optimize the yield based on the knowledge of distribution of the sparks. By HOT mechanism optimization, almost any distribution of sparks gives a power law distribution of events. There exist both numeric and analytic results for that.

One HOT mechanism is based on increasing the design degrees of freedom (DDOF). The goal is to optimize the yield, or in other words, push it towards the upper bound. The upper bound is the case where there would be no sparks and the yield would be the same as the density of the forest. The DDOF's are in this case the densities of different areas on the lattice. The lattice can be divided into smaller lattices. Each small lattice represents a design degree of freedom.

The HOT states specifically optimize yield in the presence of a constraint. A HOT state corresponds to forest, which is densely planted to maximize the timber yield, with firebreaks arranged to minimize the spread damage. The firebreaks could be of forest planted with density of the critical density. Depending on the distribution of the sparks and the optimizing method, it might be possible to plant the forest with density one everywhere else but in the small firebreaks with a critical density. In practice this means that we could have a lattice of total density 0.85 and with a yield 0.77, for example.

There are different HOT mechanisms that can be used to optimize the yield. Figures 9.10, 9.11 and 9.12 represent three different densities of forest that result, if three different methods are used in yield optimization. The assumed distribution of sparks is presented in Fig. 9.9. In the upper left corner of Fig. 9.9 the probability of sparks is almost 0.2, but in the other corners of the lattice it is lower than 10^{-10}. In the middle the probability decreases from left upper corner towards right lower corner as can be seen.

The three design mechanisms in Figs. 9.10 - 9.12 are called "grid", "evolved" and "DDOF". In grid design, the firebreaks are straight lines and the density of the forest between the firebreaks is constant (in the small quadrangles). The evolved mechanism produces such forest density that almost everywhere the density is one. Only in the upper left corner of the lattice lower densities are used. The DDOF mechanism produces as well a very dense forest. Even the firebreaks are planted with a critical density and everywhere else the density is one. The design problem in the DDOF is to optimize the areas

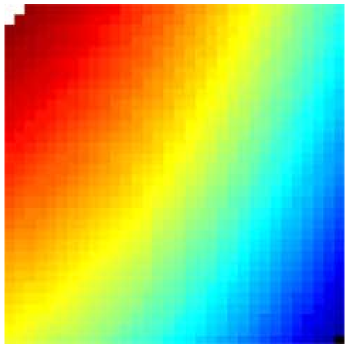

Figure 9.9: An example distribution of sparks. [15]

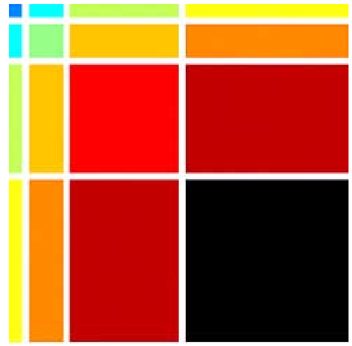

Figure 9.10: Grid. [15]

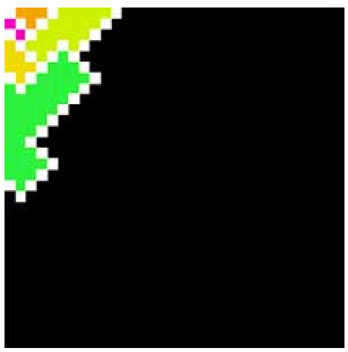

Figure 9.11: Evolved. [15]

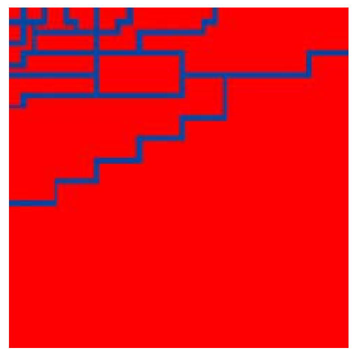

Figure 9.12: DDOF. [15]

and alignment of the firebreaks.

Increasing the design degrees of freedom increases densities and yield, because the forest can be planted densely and the firebreaks effectively stop the possible fires. Hence losses are decreased as well. Anyway, the sensitivity for design flaws increases significantly. That is why the HOT systems are said to be robust, but yet fragile. They are robust in that sense that they effectively handle disturbance situations they were designed for, but on the other hand, the systems become very sensitive for the disturbances they were not designed to handle. In the forest case, the system is very fragile, if a rare event happens i.e. a spark hits the right lower corner of the lattice.

HOT may be a unifying perspective for many systems. HOT states are both robust and fragile. They are ultimately sensitive for design flaws. Complex systems in engineering and biology are dominated by robustness tradeoffs, which result in both high performance and new sensitivities to perturbations the system was not designed to handle. The real work with HOT is in new Internet protocol design (optimizing the throughput of a network by operating in HOT state), forest fire suppression, ecosystem management, analysis of biological regulatory networks and convergent networking protocols [15].

9.5 Conclusions

In this chapter the emphasis has been on complexity. It has been approached from different directions and some essential features and terminology have been discussed. Catastrophe-, chaos- and complexity theory all have common features, but different approaches. Complexity adds dimensions to chaos. All these branches have a common problem, lack of useful applications. With complexity there is still hope...

Self-organized criticality was discussed and it refers to tendency of large dissipative systems to drive themselves into a critical state. Coupled systems may collapse because of an avalanche that results from a perturbation to the system. As an example of SOC a sandpile model (although not a very realistic one) was discussed.

Edge of chaos is the border area where the interesting phenomena (i.e. complexity) arise. The edge lies between periodic and chaotic behavior. When cellular automata are concerned the fraction of rules that lead to "alive" state determine so called λ value. With some values of λ static behavior can be detected and with some other values chaotic behavior is seen. Somewhere in between with some critical value of λ complexity arises. That value of λ represents the edge of chaos.

Highly optimized tolerance (HOT) was discussed in the last section. The idea is to optimize the profit, yield or throughput of a complex system. By design we can reduce the risk of catastrophes, but the resulting systems operating in HOT states with considerable performance are yet fragile.

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