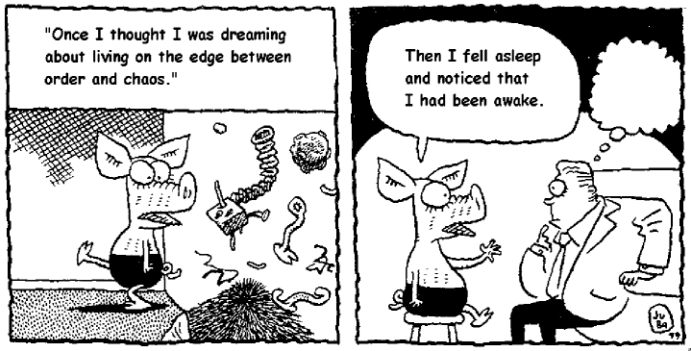
AS-74.4192 Elementary Cybernetics

Lecture 2:

Research on Complex Systems



HELSINKI UNIVERSITY OF TECHNOLOGY
Department of Automation and Systems Technology
Cybernetics Group

Philosopher's Stone of Today?

- J. Holland:

  "Many of our most troubling long-range problems – trade imbalances, sustainability, AIDS, genetic defects, mental health, computer viruses – center on certain systems of extraordinary complexity. The systems that host these problems – economies, ecologies, immune systems, embryos, nervous systems, computer networks – appear to be as diverse as the problems. Despite appearances, however, the systems do share characteristics ... This is more than terminology. It signals our intuition that there are general principles that govern all cas behavior, principles that point to ways of solving the attendant problems. ..."

Gallup

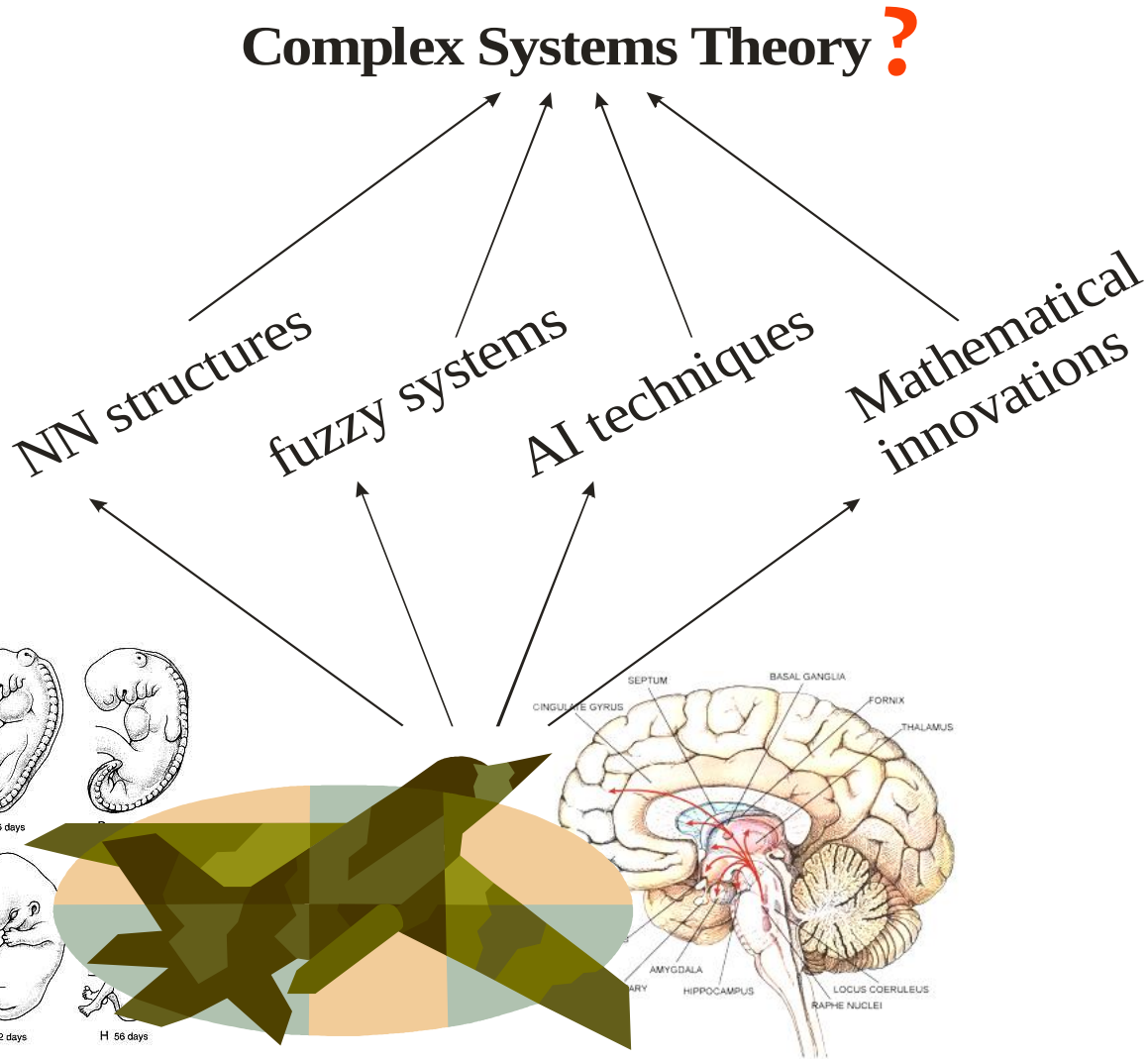
What do you know about *alchemy*?

Do you laugh at alchemists?

(remember Newton ...)

Credo

- "Clearly, there is something special about complex systems"
- *Truly*? Do you think so?



“Negativism” vs. positivism

- “Key theories are already there, one only needs to fill in the gaps”
- K. Enqvist (etc.): Everything is energy – one only needs to write the Hamiltonians ... Nonlinearities *of course* then result in observed illusion of complexity
- And *of course*, these system-specific energy expressions are extremely complicated ...

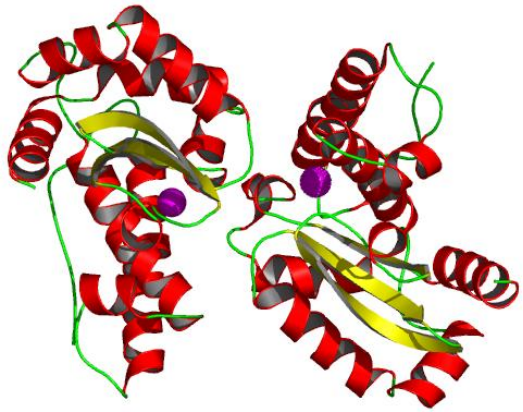
Something to ponder

- Aristotle : “Heart is the home of soul”
  - Heart is in the “innermost” organ
  - Speech comes from the chest, where the heart is
  - Heartbeat accelerates when one is excited, etc.
  - Brain is only needed for cooling of blood!
- Aristotle was the big authority for more than 1000 years, offering the most logical explanations at that time
  - Before gravitation law, based on the Aristotelian world view, the best explanations based on flat Earth hypothesis (objects want to fall “down”)
- Further: Before the theory of relativity, the best explanation for diversity of species *was* divine (there is not enough coal in the Sun to last for millions of years)

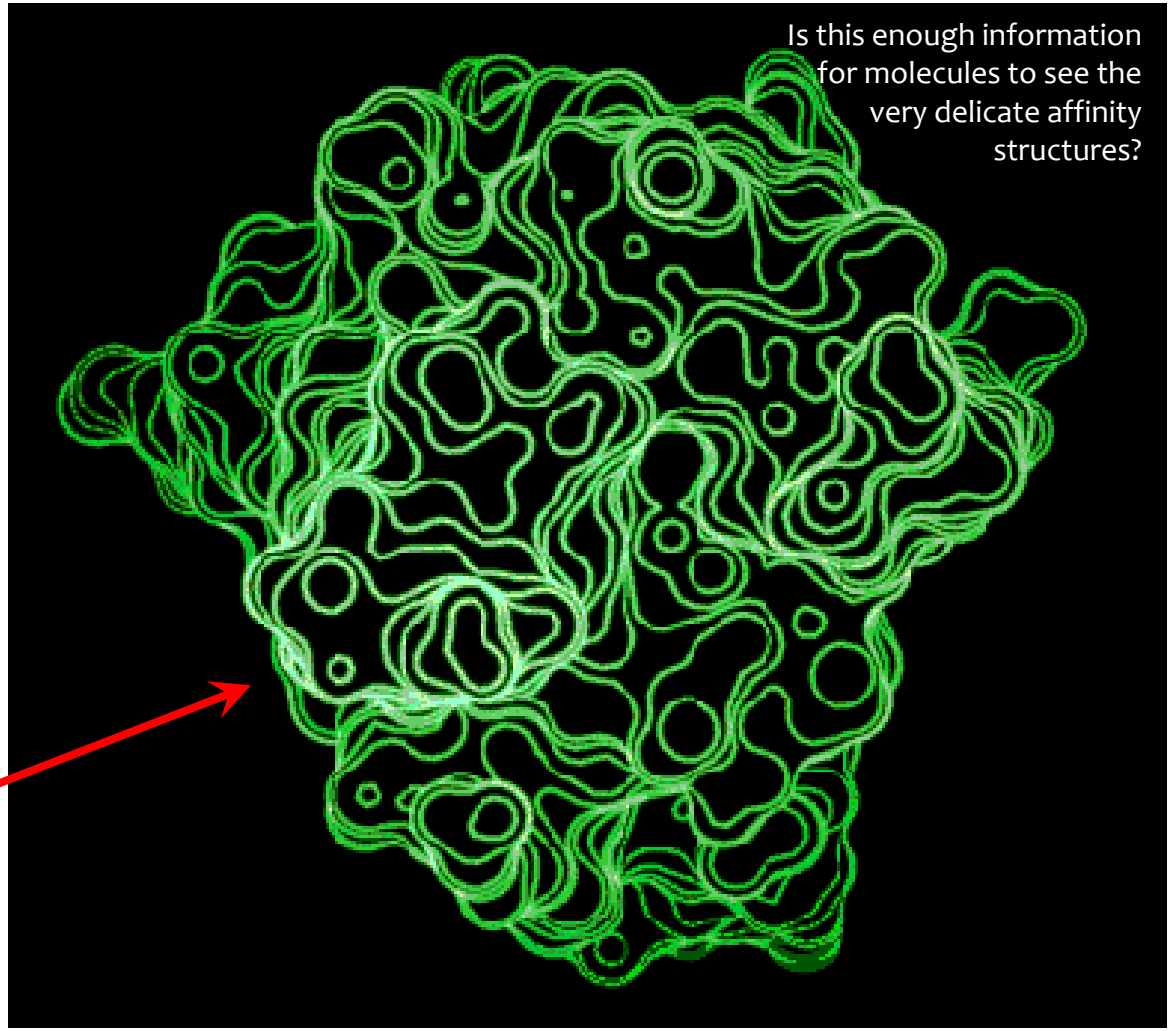
- One's thinking is bound to one's own world view; are we now on the correct track?
- Thinking patterns 500 years ago seem so ridiculous – what do they think about *us* 500 years from now in the future?
- Today there are so many new incompatible observations that one can say that there are more mysteries than ever before
- The "best explanations" are probably to be changed again
- Evidence & explanations are not yet in balance – examples:
  - *Gene transcription + translation* – intelligence needed in coordination!?
  - *Proteins + enzymes* – huge number of functionalities: Pattern recognition?!
  - How to understand and model *protein folding*?
  - What is the nature of *orbitals*, the predestinated structures in molecules?

Example #1

- For example: Enzyme *superoxide dismutase*



- Only electric fields can be experienced by other molecules



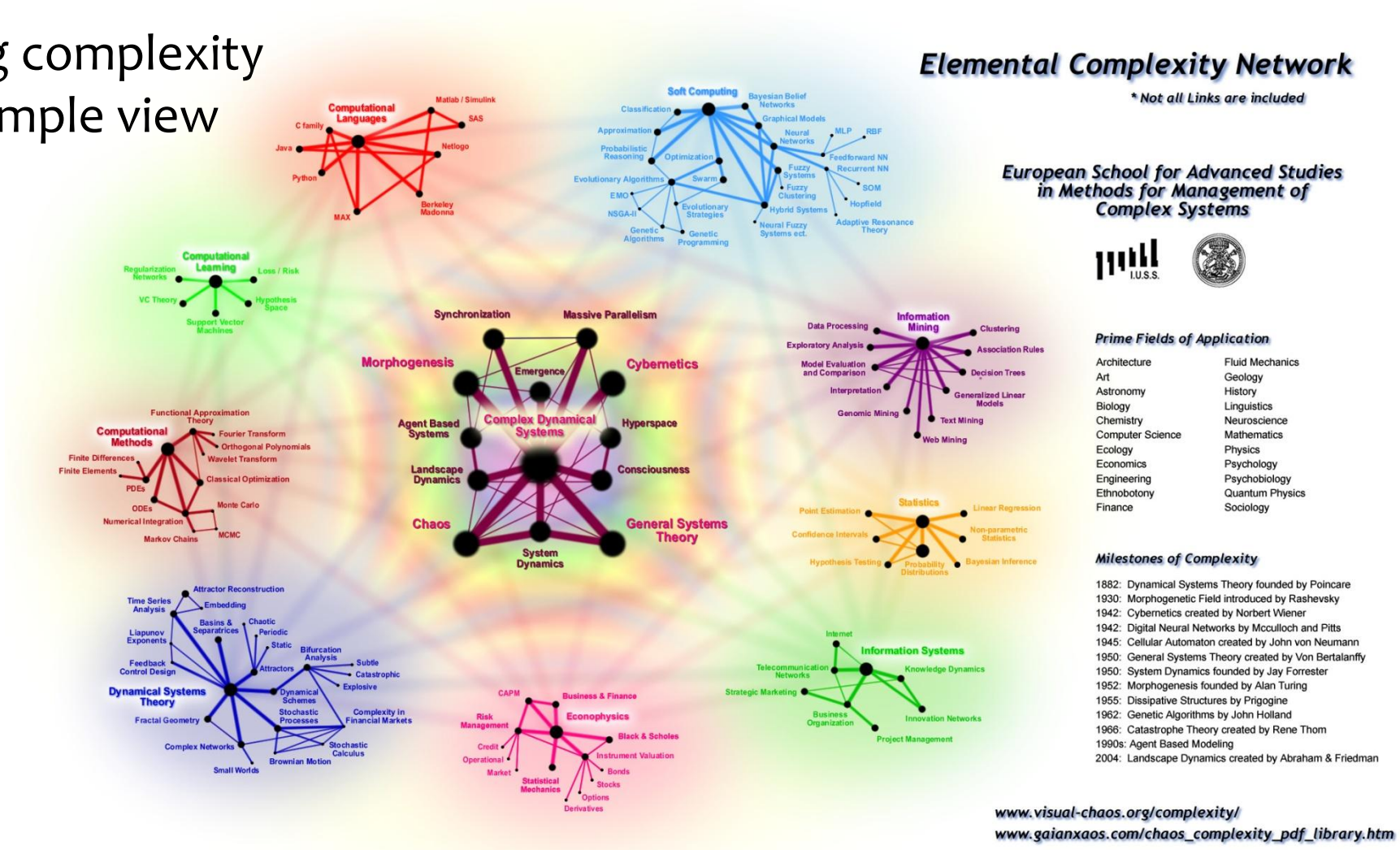
Example #2

- How to explain the symmetricity in snow crystals?
- Does there exist some internal communication?
- Today's explanation: "All parts of a single snowflake experience exactly the same environmental parameters"
- However – clearly, different parts are NOT in the same phase of development

Claim: more analysis truly IS needed ...!

Complexity – how to attack it?

Mapping complexity
– an example view



An age-old challenge indeed

- Traditional way to tackle with complexity: Construct *hierarchies*, study levels reductionistically one at a time
- Natural approach for humans + also "natural for nature"?
  - Aristotle, Linné: Taxonomies (Systematic but not systemic!)
  - H. Simon (1969): "Architecture of Complex Systems" – *robustness*
  - Correspondingly in large-scale industrial systems: Hierarchical control
- However, fixed hierarchies cannot capture *emergent phenomena* – the essence of complex systems
  - How to define *intelligence*?
  - How to define *life*?
  - How to define *robustness*?
- Something new is needed ...

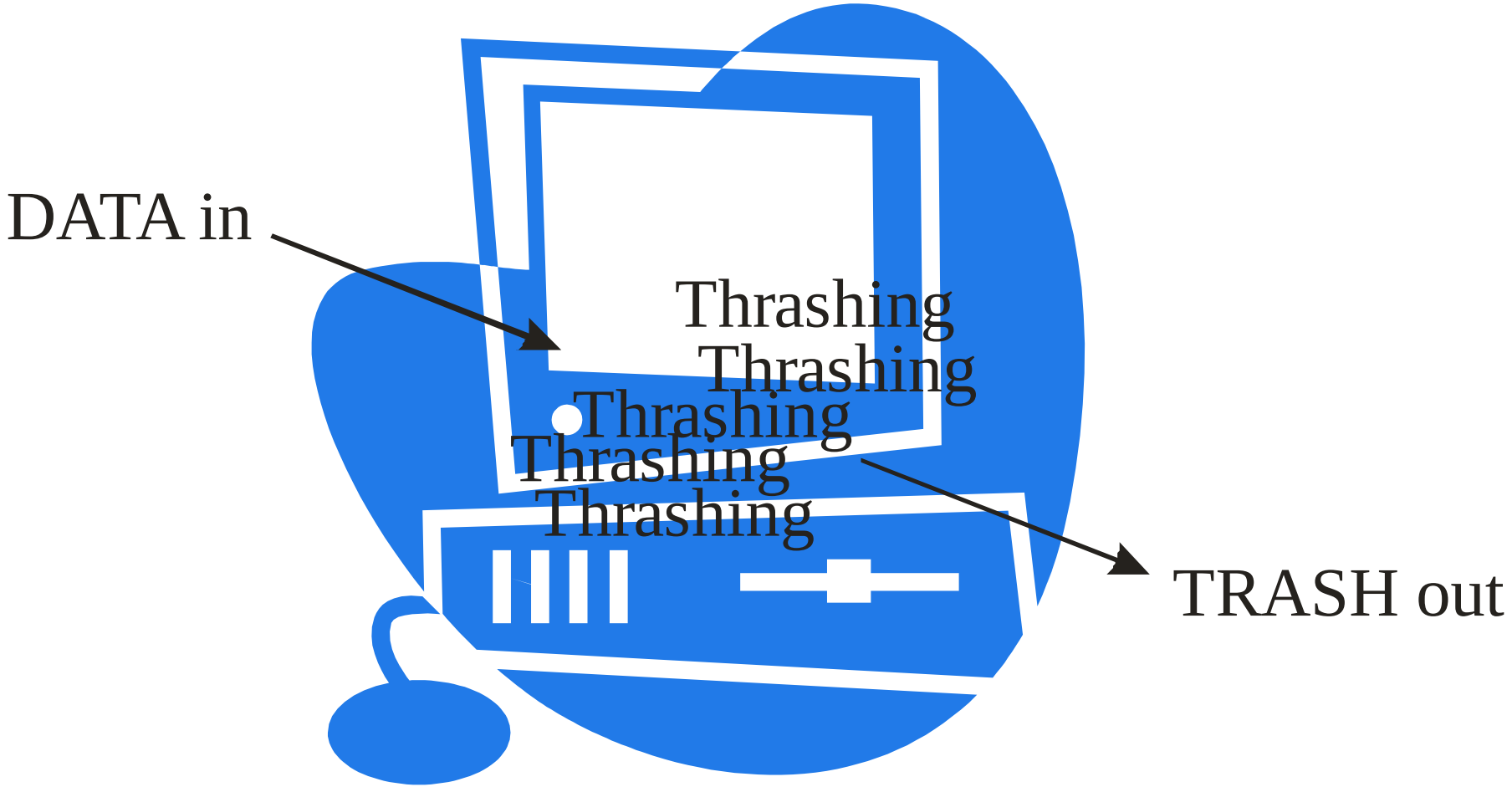
- Now: Contemporary approaches to seeing complex systems

Emergence with Computation?

- Challenge: Complex systems are characterized by *emergent* properties – "the whole is more than the sum of the parts"
- How to master something that by definition defies reductionistic analysis attempts?
- New kind of thinking ("non-Greek"!) is needed ...

- *Computationalism* promises escape from the deadlock?
- Trust in thrust of computing: "In 20 years, computer will be more intelligent than a human"
- But it is not only computing power that is needed; how can computation make non-trivial phenomena emerge?
- Conceptual tools + rigor needed

... Otherwise ...


DATA in
Thrashing
Thrashing
Thrashing
Thrashing
Thrashing
TRASH out

Background: Chaos "theory"

- ... Would never have been discovered without computer!?
- Observation: Very simple (nonlinear) functions, when iterated, result in very complex-looking forms
- For example, study the simplest possible (?) discrete-time constrained growth model (logistic model):

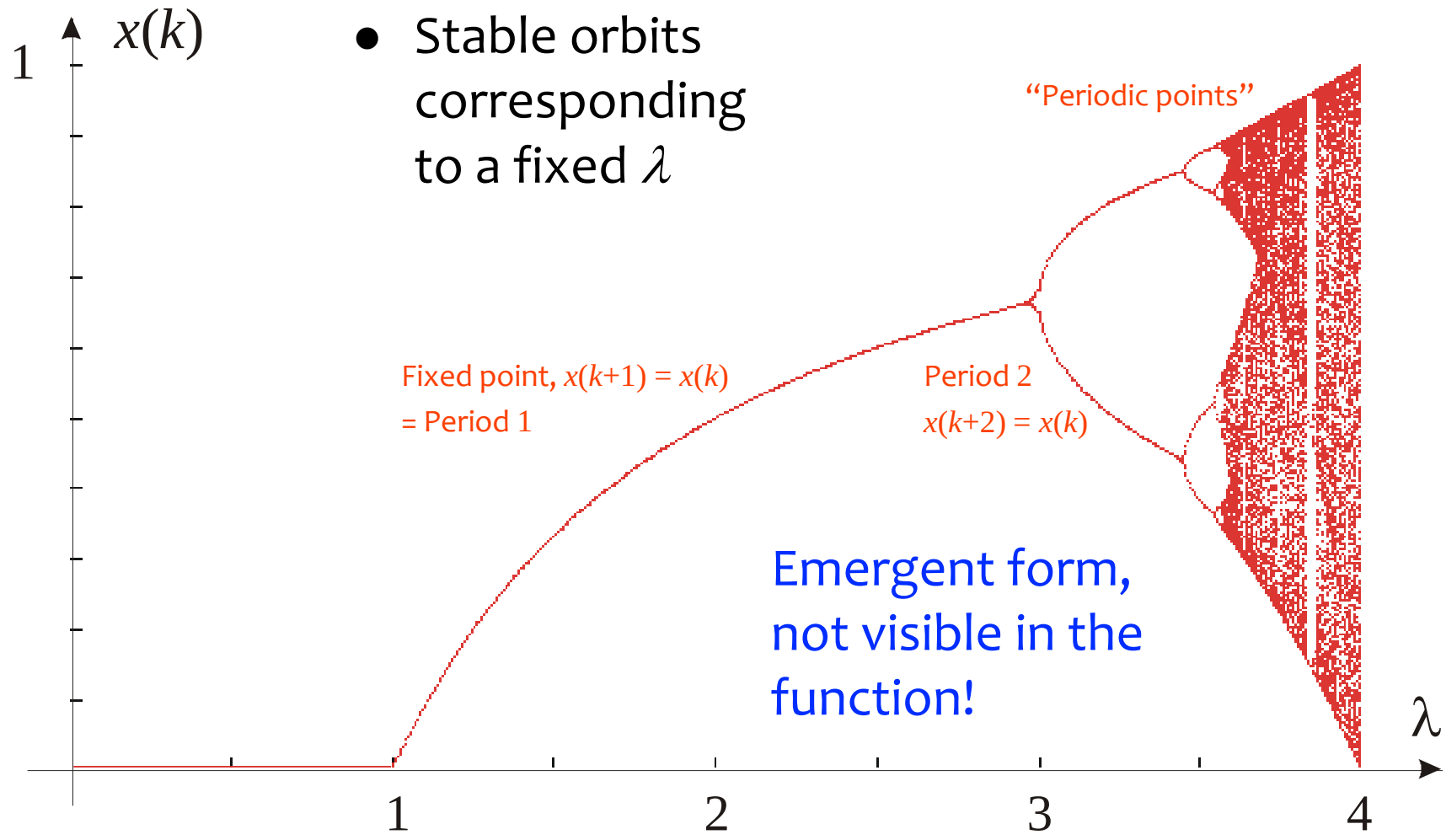
$$x(k+1) = \lambda x(k) \cdot \big(1 - x(k)\big)$$
$$= \lambda x(k) - \lambda x^2(k).$$

Power of feedback reinvented!

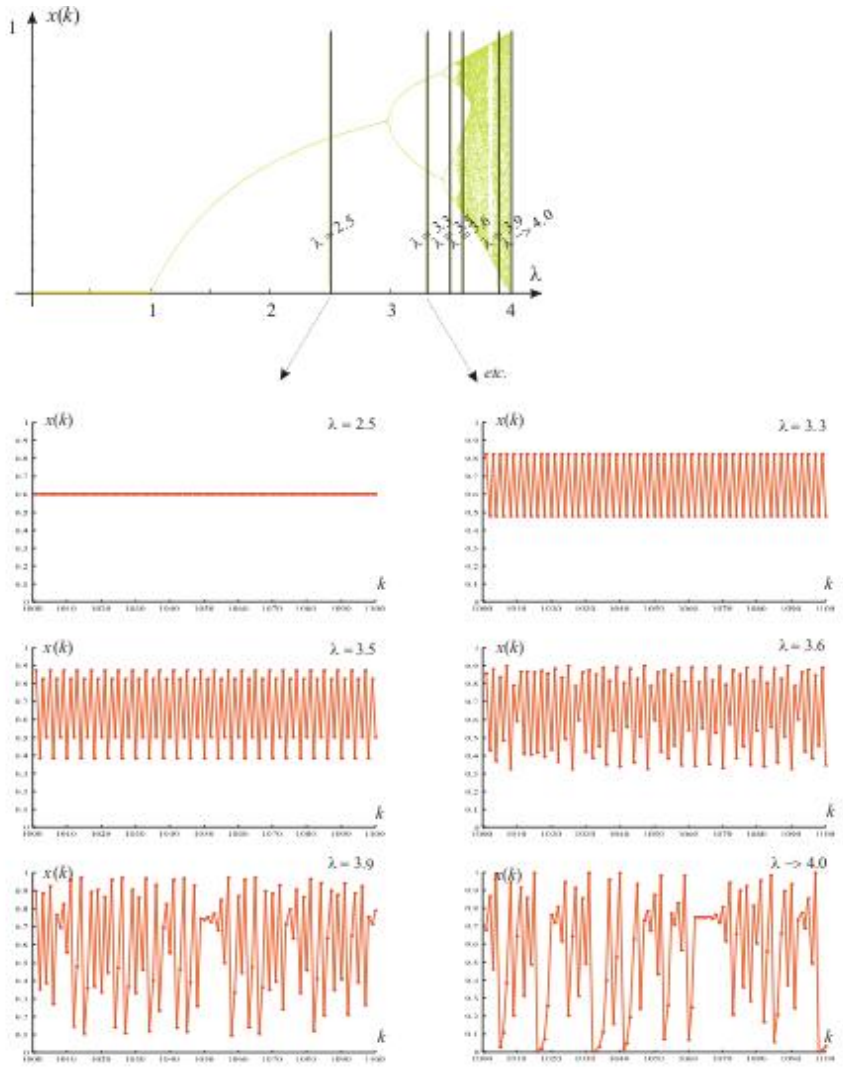
- Linear term: Exponential growth if no constraints
- Quadratic term: Inverse effect if population is too large
- Parameter λ is the *growth factor*

“Bifurcation diagram”

- Stable orbits corresponding to a fixed λ



Route to chaos



- For small $\lambda < 1$, extinction
- For $1 < \lambda < 3$, steady state
- After that, *doubling* of the length of the stable orbit
- Orbit lengths 2, 4 , 8, 16, ... in order
- After that, also odd cycles; indeed, any cycle found if λ is selected appropriately
- When λ goes towards 4, cycle length goes to infinity = *chaos*

- What does this look like in higher dimension?
- For simplicity, the complex-valued iteration (a "complex complex" system?!) is defined as

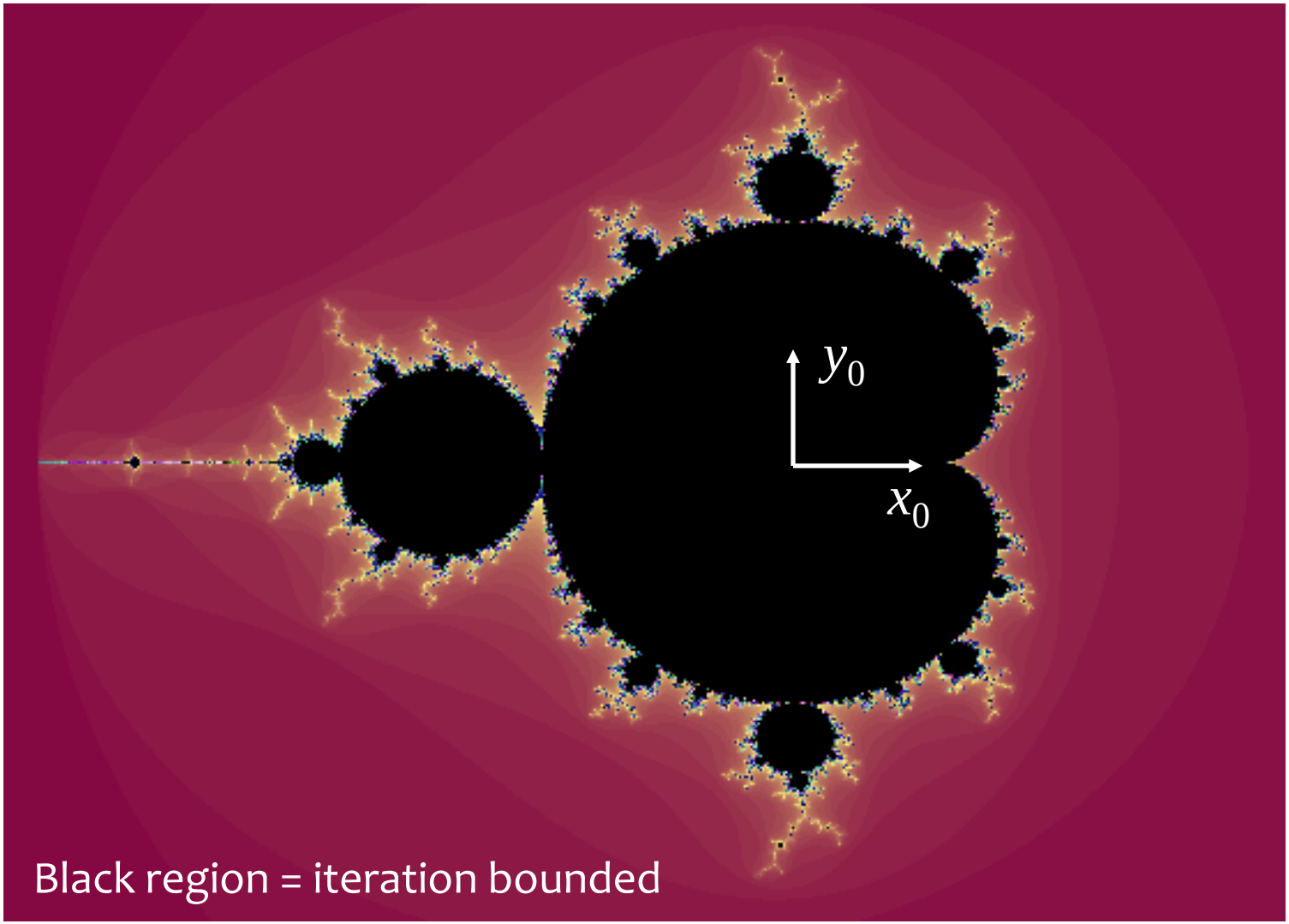
$$z(k+1) = z^2(k) + z_0$$

- Using only real variables, this can be written

$$\begin{cases} x(k+1) = x^2(k) - y^2(k) + x_0 \\ y(k+1) = 2x(k)y(k) + y_0. \end{cases}$$

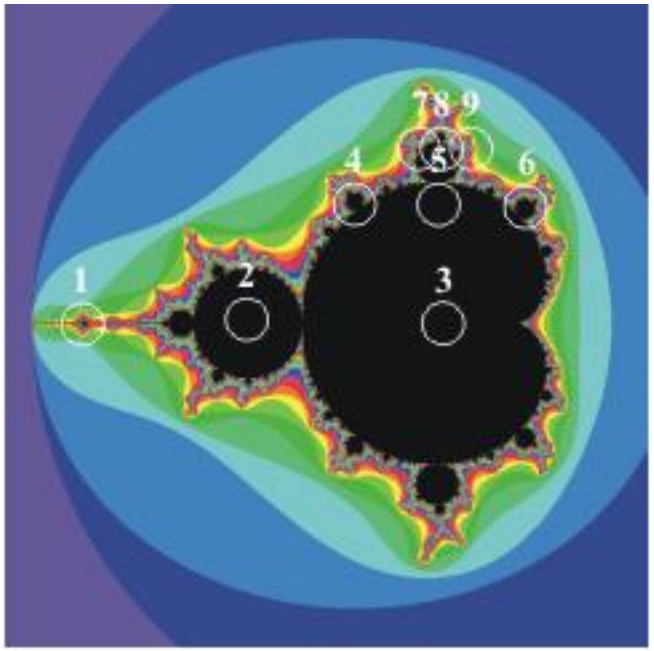
- Assuming that one selects some constants x_0 and y_0, and starts from $x(0) = y(0) = 0$, what will happen?

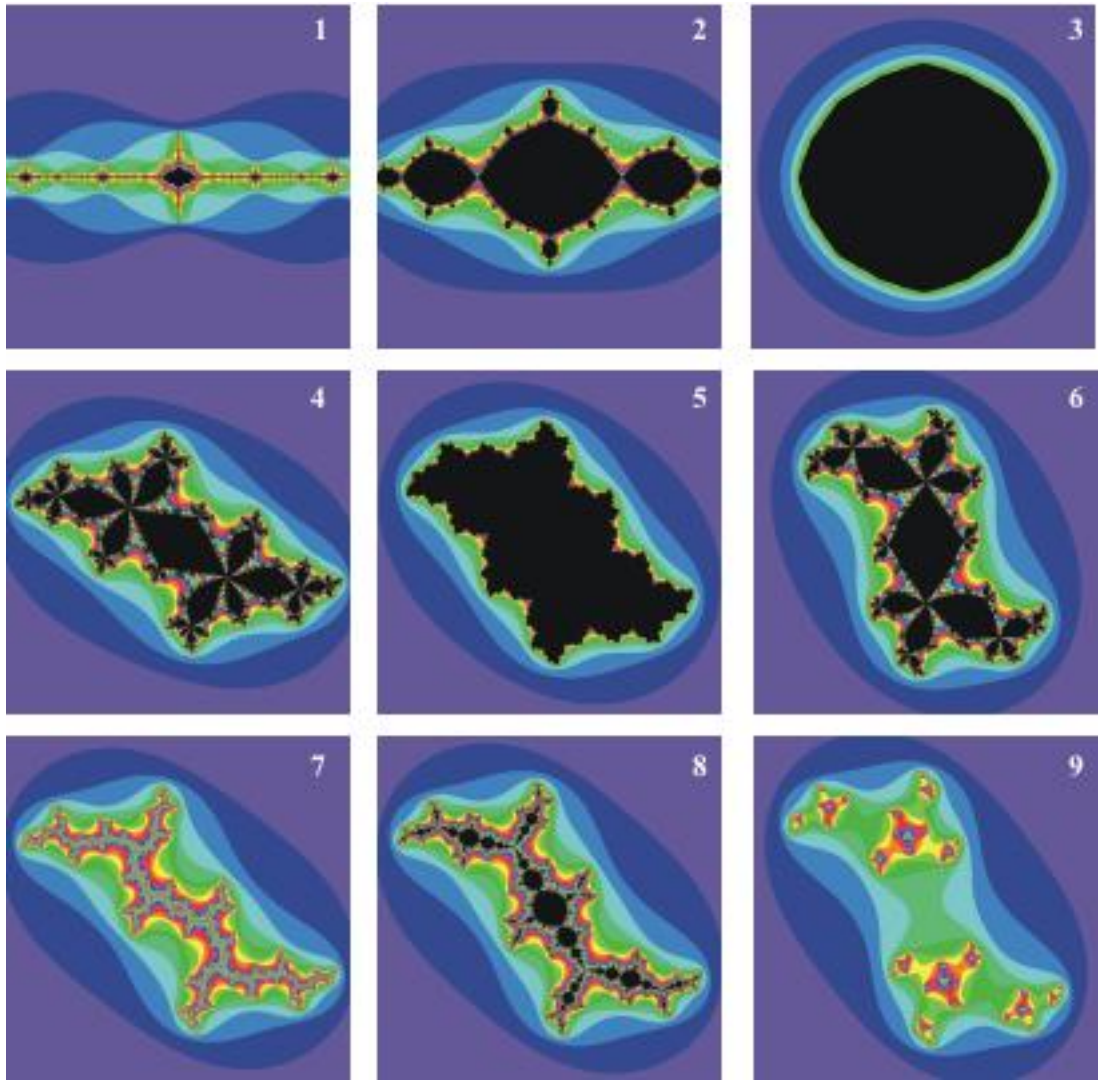
“Mandelbrot set”



“Julia sets”

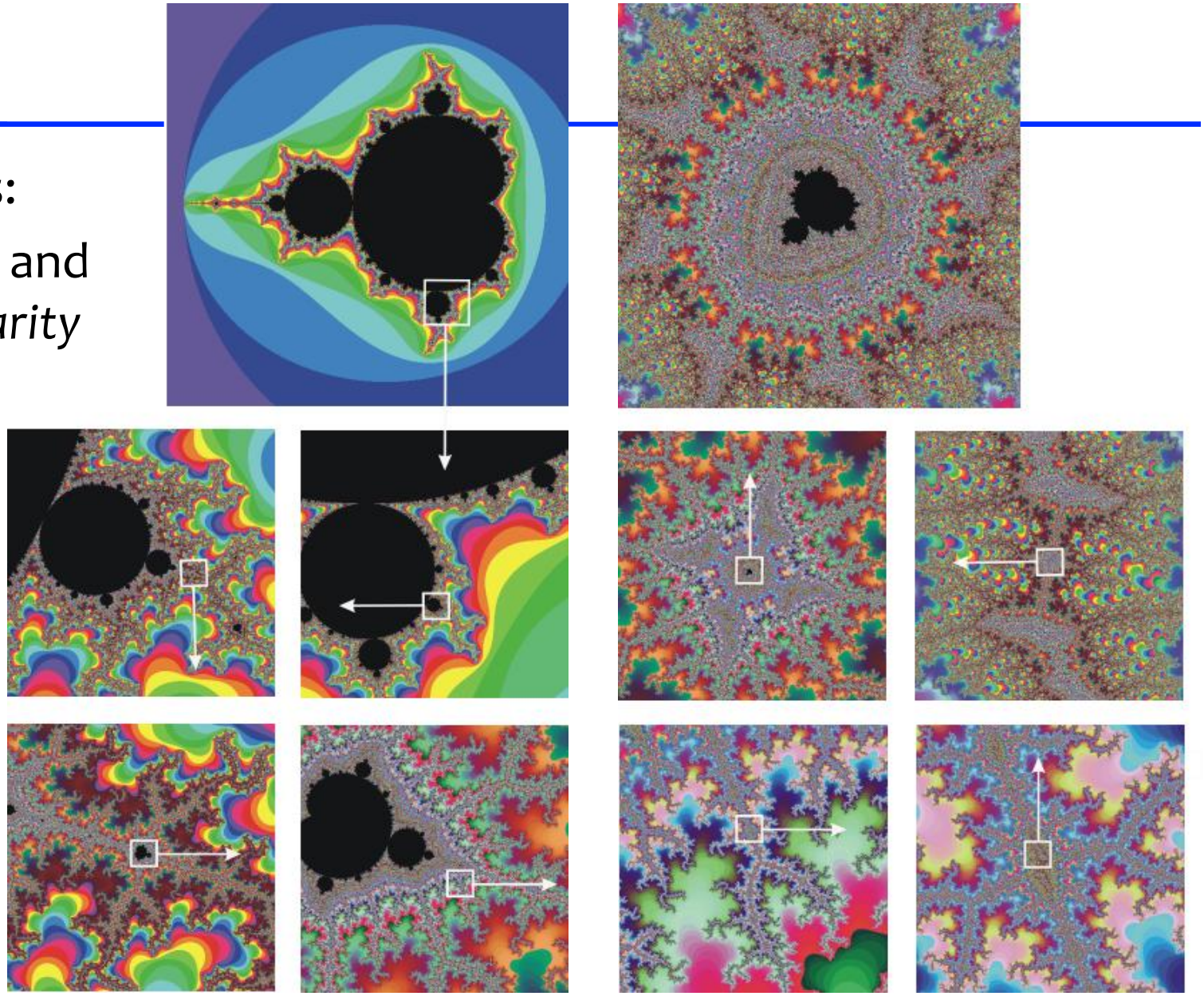
- Orbits for fixed (x_0,y_0) – as shown below – $(x(k),y(k))$ shown in black on the right



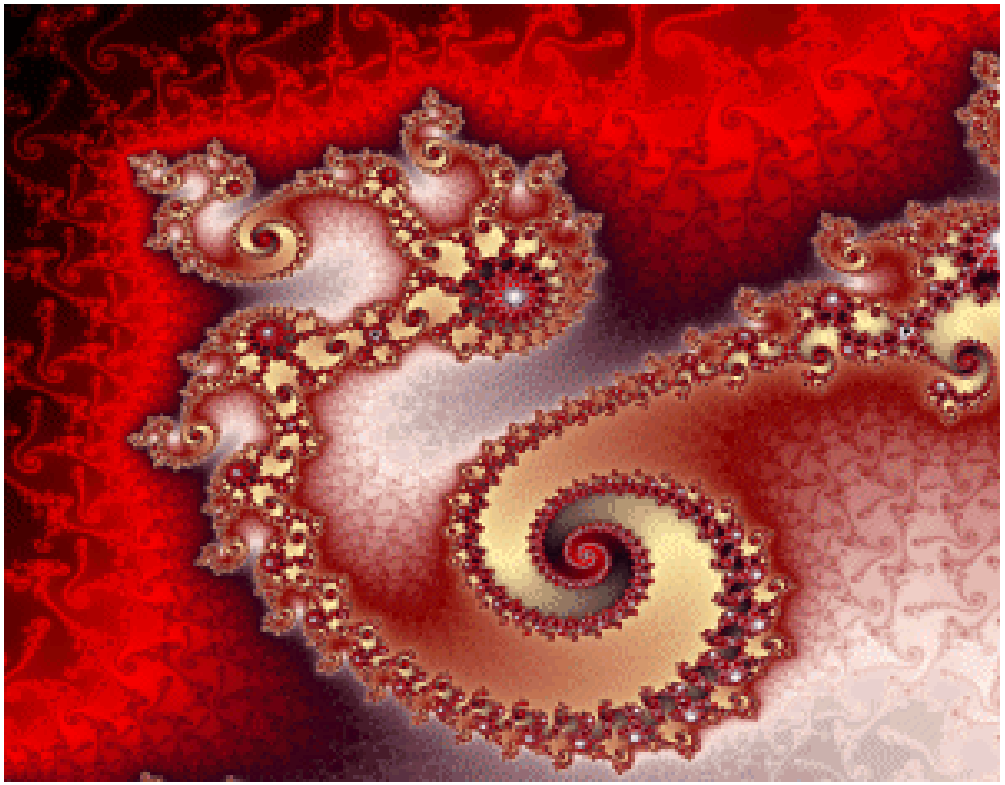


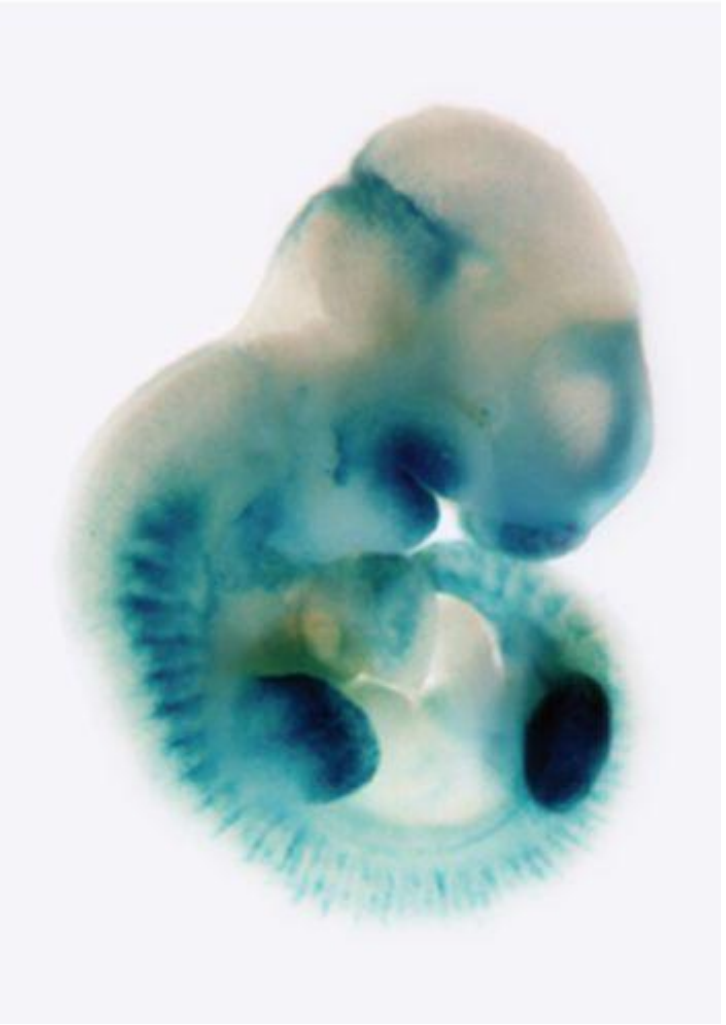
- Concepts:

  Fractality and *self-similarity*



- Now it seems we are touching the essence of complex systems!?





Dilemma: “Butterfly Effect”

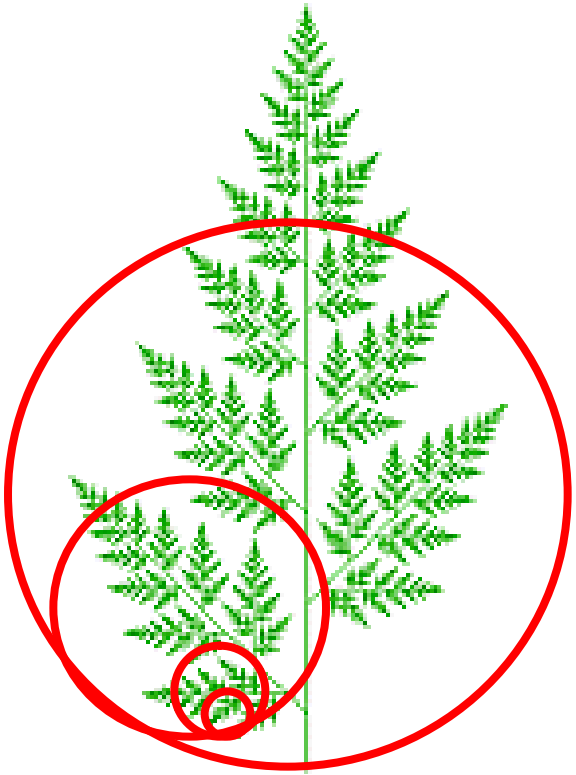
- Basic problem in chaos thinking: Chaotic models are highly sensitive to the initial conditions and parameters

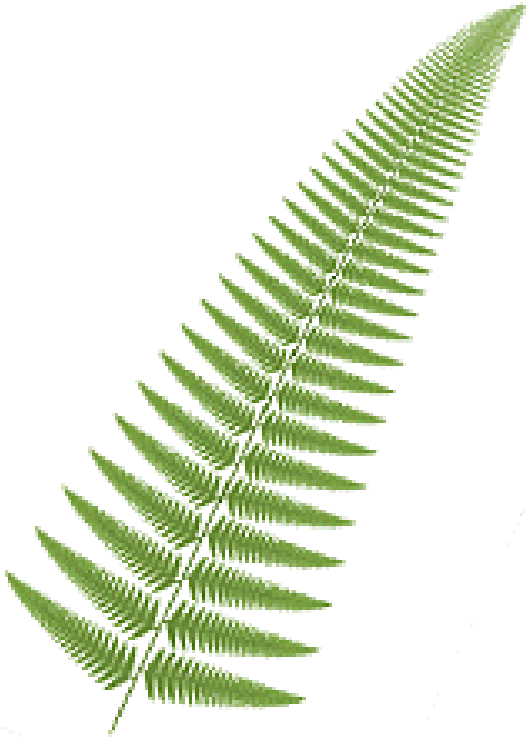
  … And there exist more systems than there can exist models!
- The models cannot then reliably simulate real systems

  … Are models of any use? *Specially – data-based models!?*

There are also convergent behaviors

- No matter where you start from, there sometimes emerge interesting self-similar patterns in iterative systems ...





From chaos to complexity theory

- Universality in nonlinear systems: The same behavioral complexity is found in many classes of nonlinearities
- Assume that Nature is based on such function iterations
- Stephen Wolfram's Theory of Everything: "Universe can be coded in four lines of Mathematica code"!
- Compare to alchemists and the Philosopher's Stone ... Similarly, one is searching for the fundamental principle
- How could the chaos process be inverted: How to find the underlying formulas beneath observed patterns?
- To elaborate on this, solid formulations are needed ...

Result of convergent iterations: *Fractals*

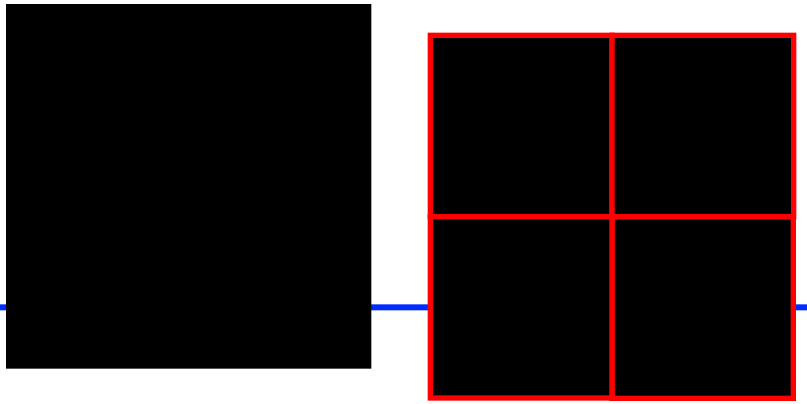
- Fractals = New framework for "data-based hierarchies"
- Similar-looking structures repeat themselves in different scales

$$\text{fractal dimension } D = \frac{\log(\text{self-similar fractions})}{\log(\text{magnification factor})}$$

- Simple examples:

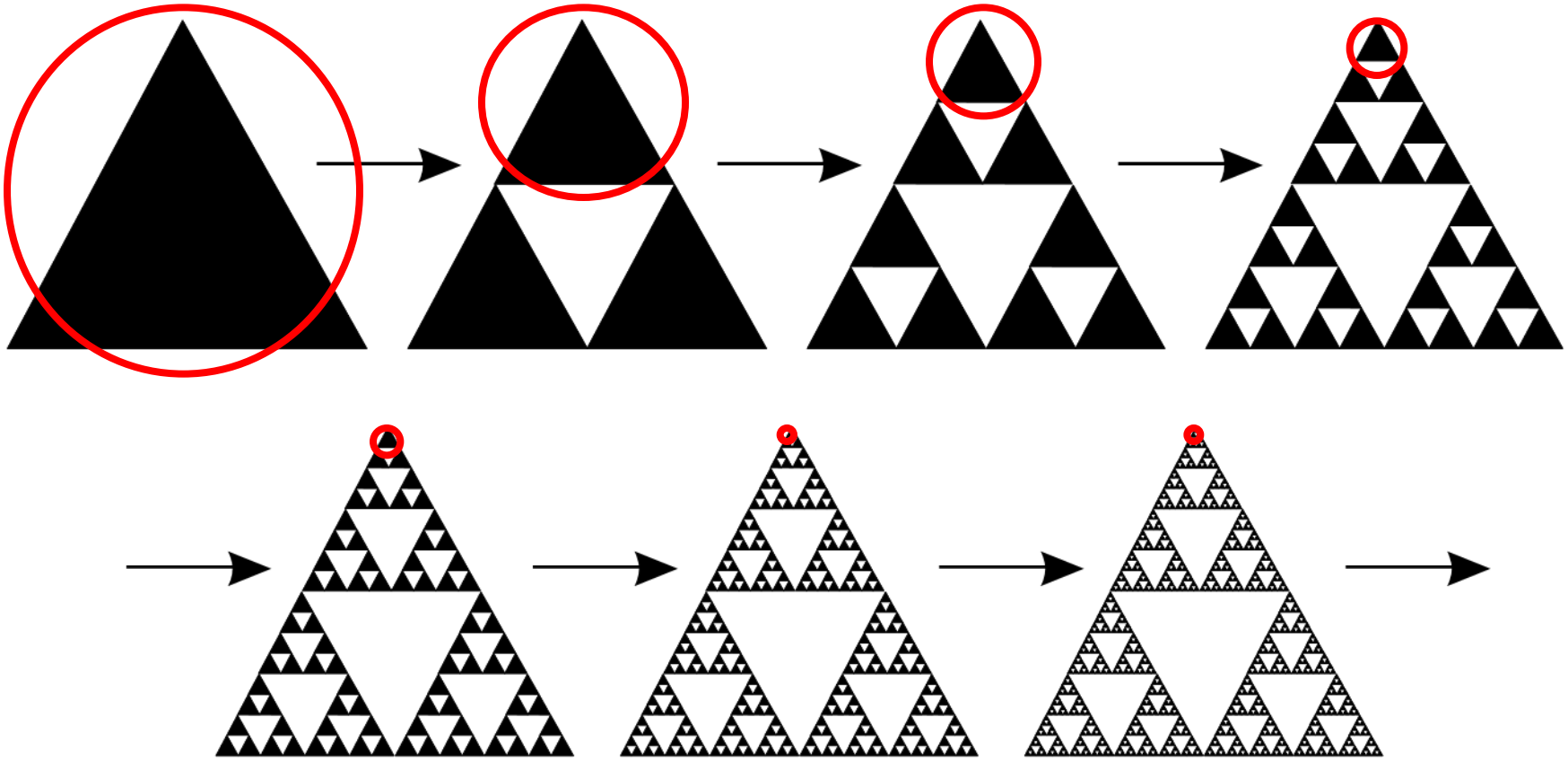
$D = \log(3)/\log(3) = 1$

$D = \log(4)/\log(2) = 2$



“Sierpinski triangle”

- Dimension $D = \log(3)/\log(2) = 1.585$



"Power law"

- Inverse look at fractality:

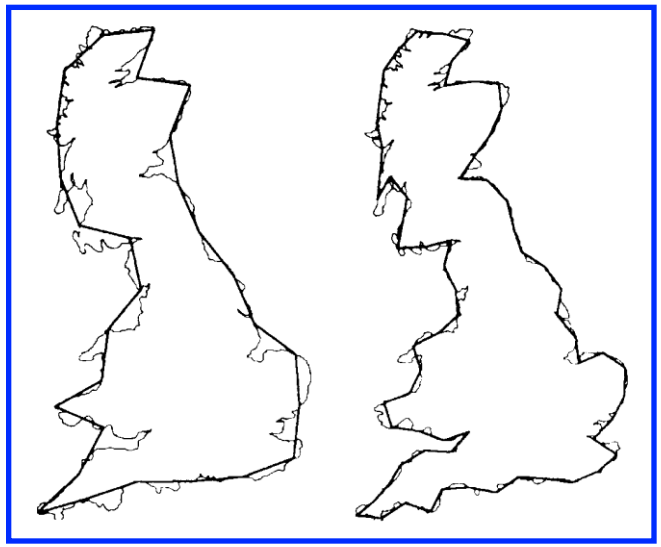
$$\log(\text{self-similar fractions}) = D \cdot \log(\text{magnification factor})$$

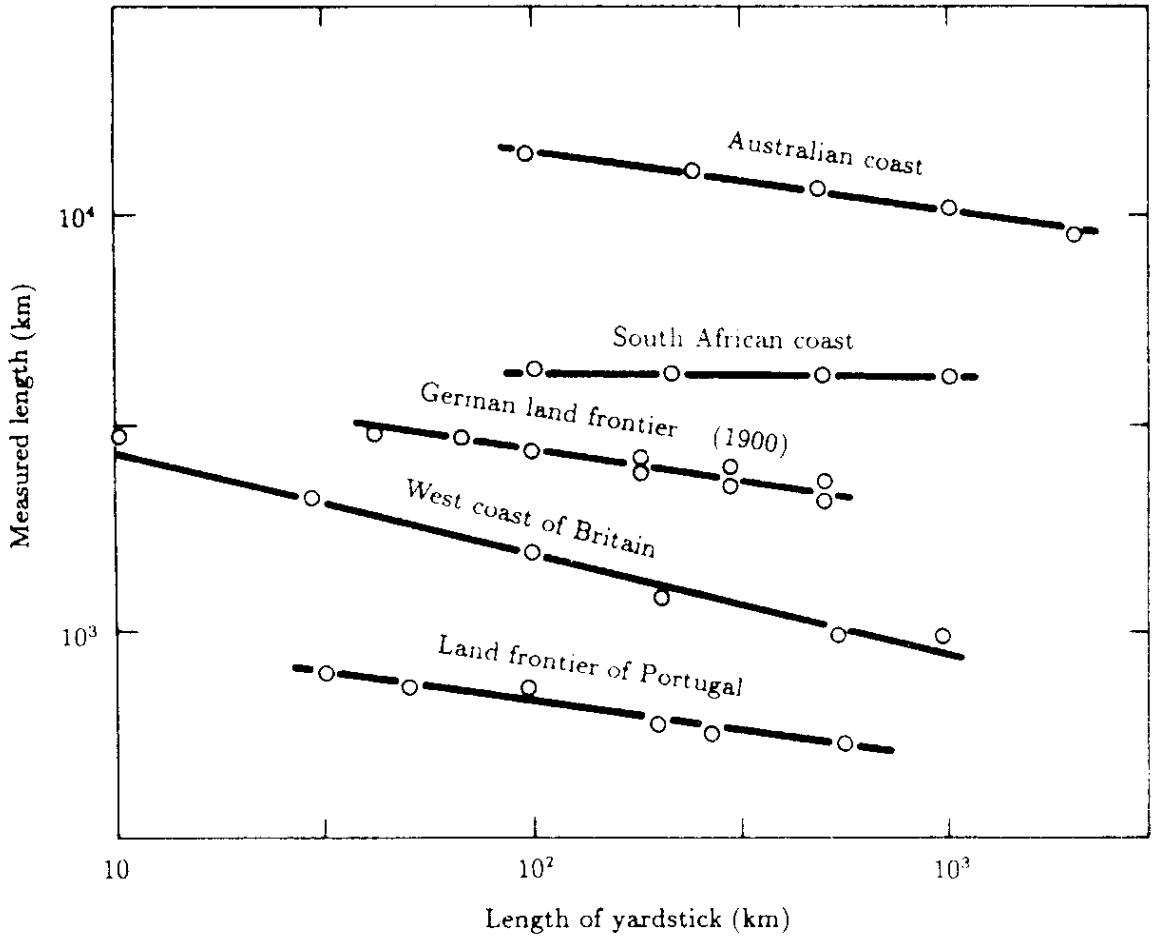
or

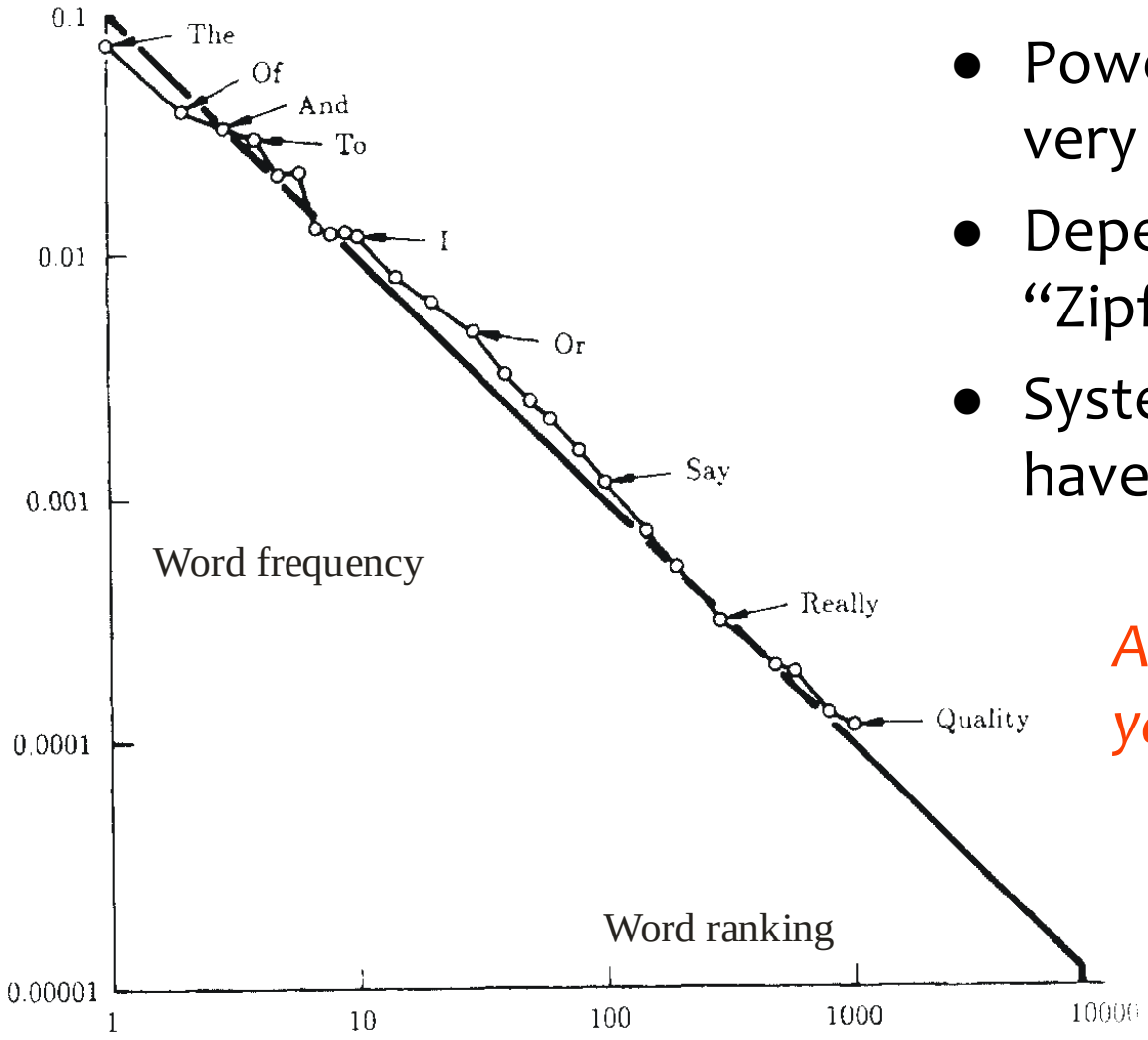
$$\text{self-similar fractions} = (\text{magnification factor})^{D}$$

- Fractality is manifested as linearity on the log/log scale = *power law*
- Rate of growth/decay = fractal dimension
- Offers a practical way to analyze existing systems
  – and power law is observed in very different environments!

- Natural formations follow power law
- For example, fractalities of coastlines:







- Power law is observed also in very abstract systems
- Dependency known also as "Zipf's law"
- Systems (for some reason) have been *self-organized*

Applicable to any kind of yardstick + quantity!?

Theories and buzzwords

edge of chaos

scale invariance

phase transitions

critical exponents

inverse-square law

Hausdorff dimension

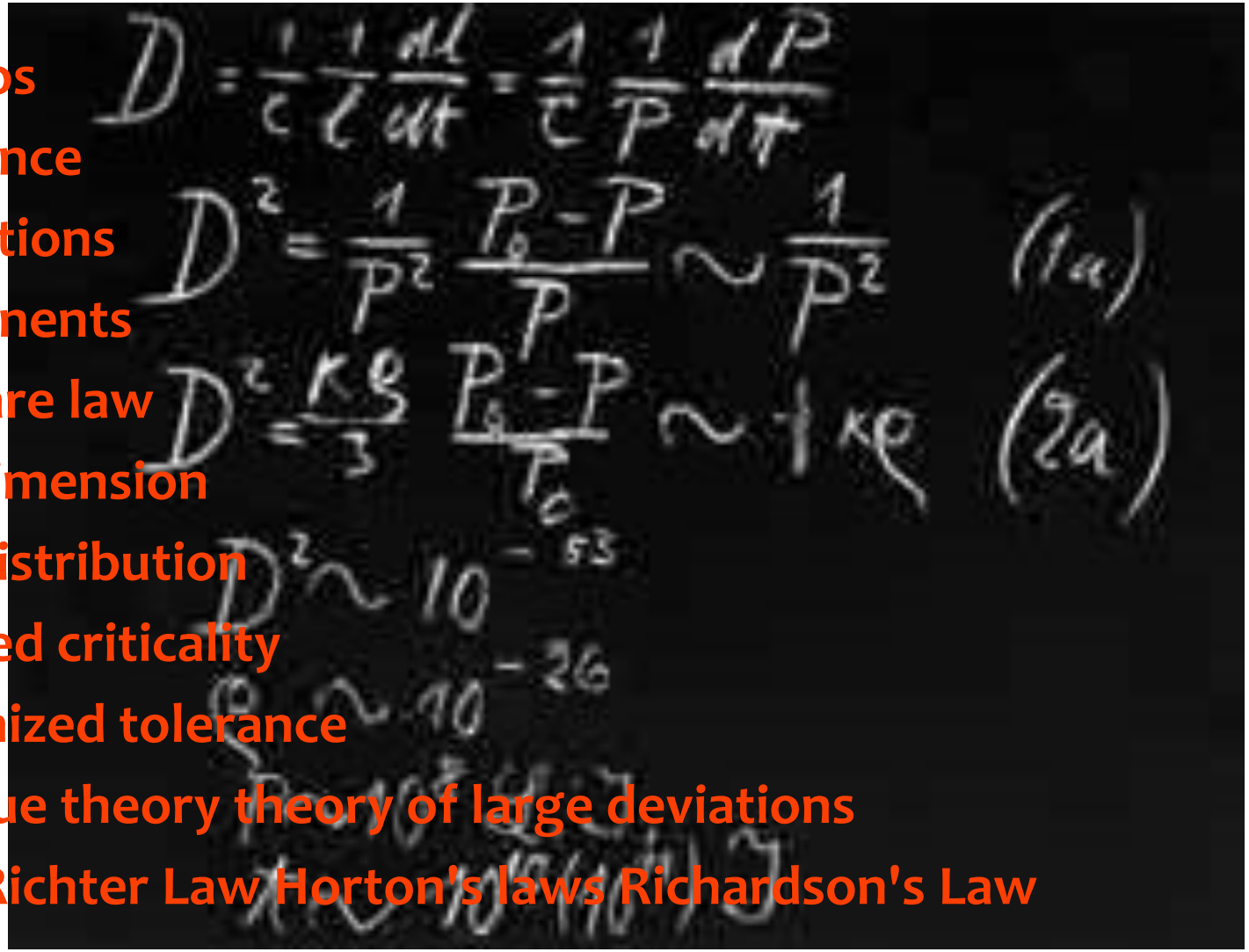
lognormal distribution

self-organized criticality

highly optimized tolerance

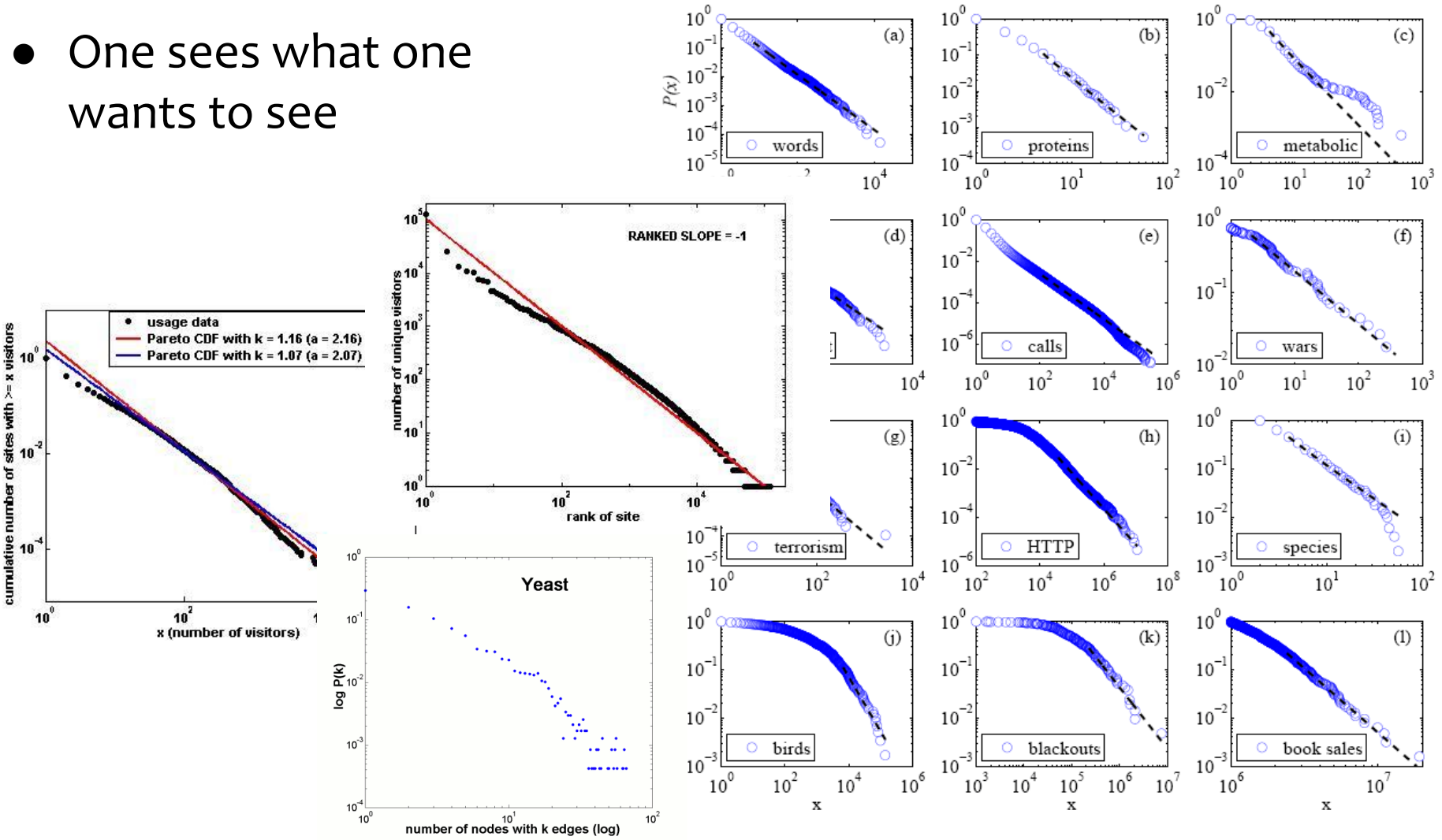
extreme value theory theory of large deviations

Gutenberg-Richter Law Horton's laws Richardson's Law



… However, to be quite honest …

- One sees what one wants to see



Wikipedia: … random fractals can be used to describe many highly irregular real-world objects. Other applications of fractals include:

- Classification of histopathology slides in medicine
- Fractal landscape or Coastline complexity
- Enzyme/enzymology (Michaelis-Menten kinetics)
- Generation of new music
- Generation of various art forms
- Signal and image compression
- Seismology
- Fractal in Soil Mechanics
- Computer and video game design, especially computer graphics for organic environments and as part of procedural generation
- Fractography and fracture mechanics
- Fractal antennas — Small size antennas using fractal shapes
- Small angle scattering theory of fractally rough systems
- Neo-hippies t-shirts and other fashion
- Generation of patterns for camouflage, such as MARPAT
- Digital sundial
- Generation of Price Series

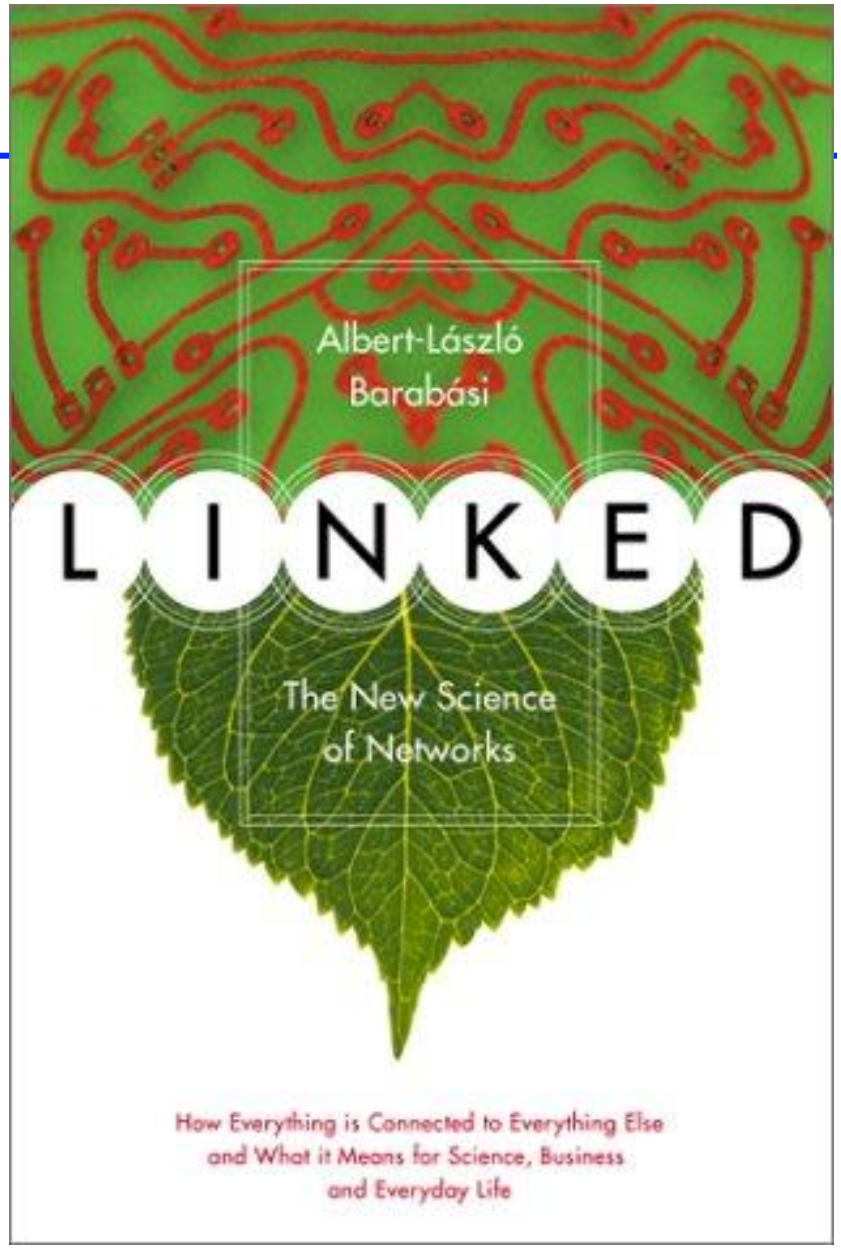
Hmmm ... remember, for example, that r and F also follow power law!

$$F = \gamma \frac{m_1 m_2}{r^2}$$

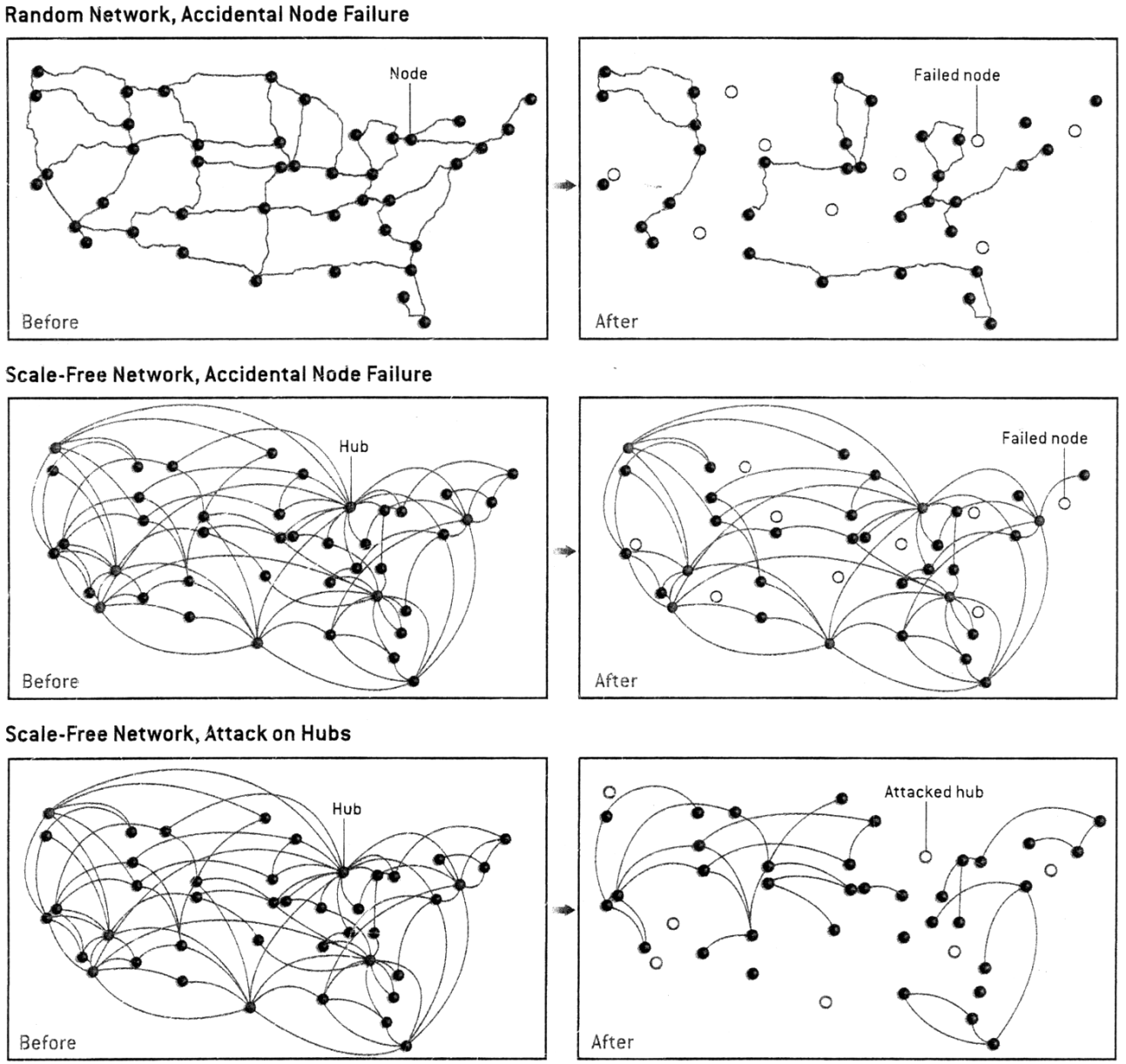
Another starting point

- Albert-László Barabási: Everything is linked and part of a network
- Result:

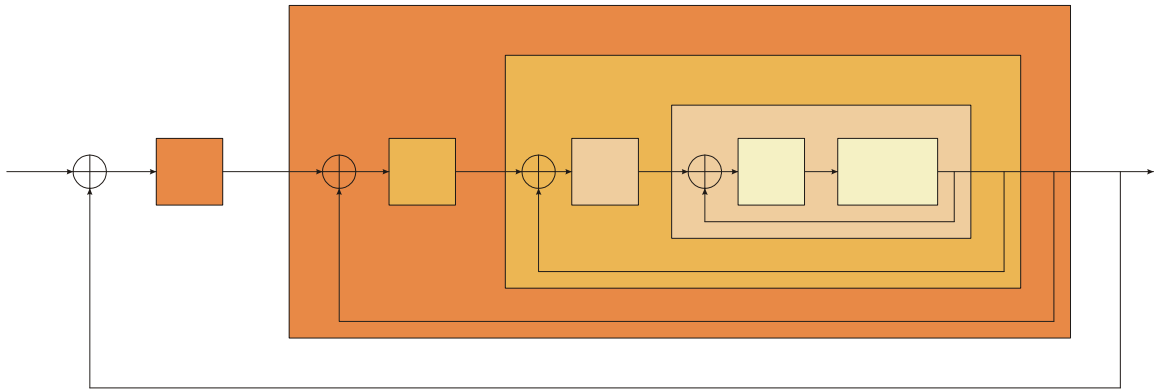
  Networks follow power law



- Motivation for fractal structure: increased robustness



- A complex (cascade) control system can also be assumed to be a fractal construction
  - Innermost structures: Stabilizing controls (hundreds)
  - Next levels: Regulatory controls (dozens)
  - Highest levels: Production control and optimization (few)
- Fractality ideas directly applicable in practice?



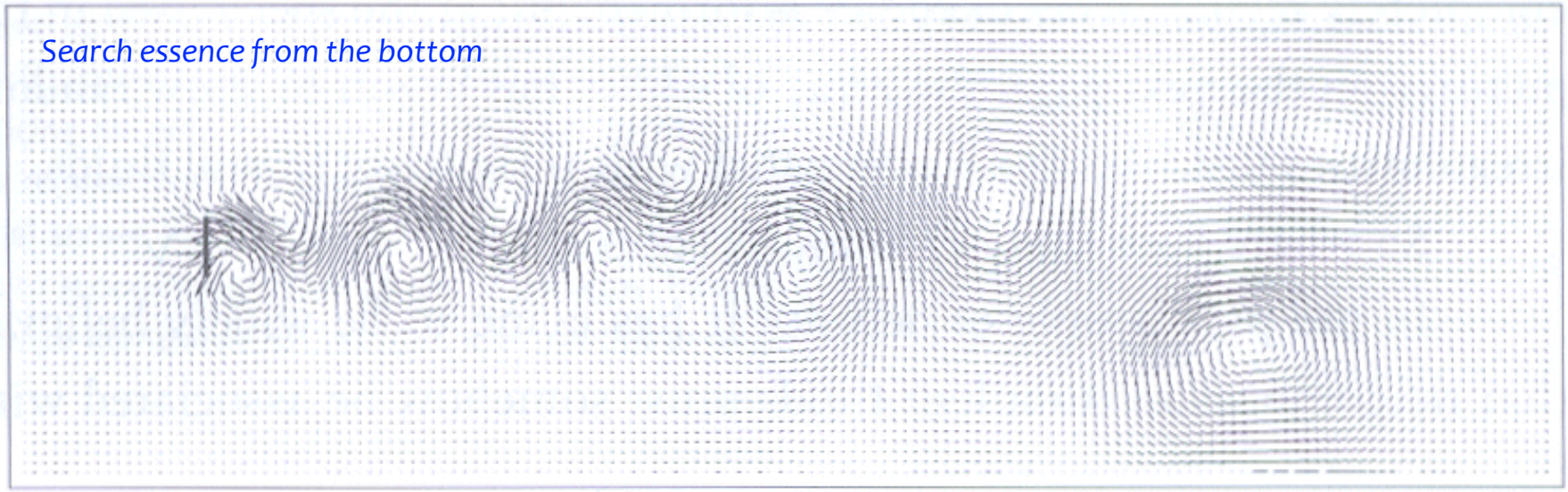
Compare slopes to Bode diagrams ...?

- If nature has in its evolutionary optimization processes arrived at fractal designs, why not directly and explicitly imitate the ideas?
- However, fractal theory is not compatible with the existing control engineering paradigm: Traditions are very different
- And, after all, there are no ready-to-use tools for control engineering tasks
- Fractal theory is better for analysis (studying existing structures) than for synthesis (design of new ones)

Are there any other available approaches?

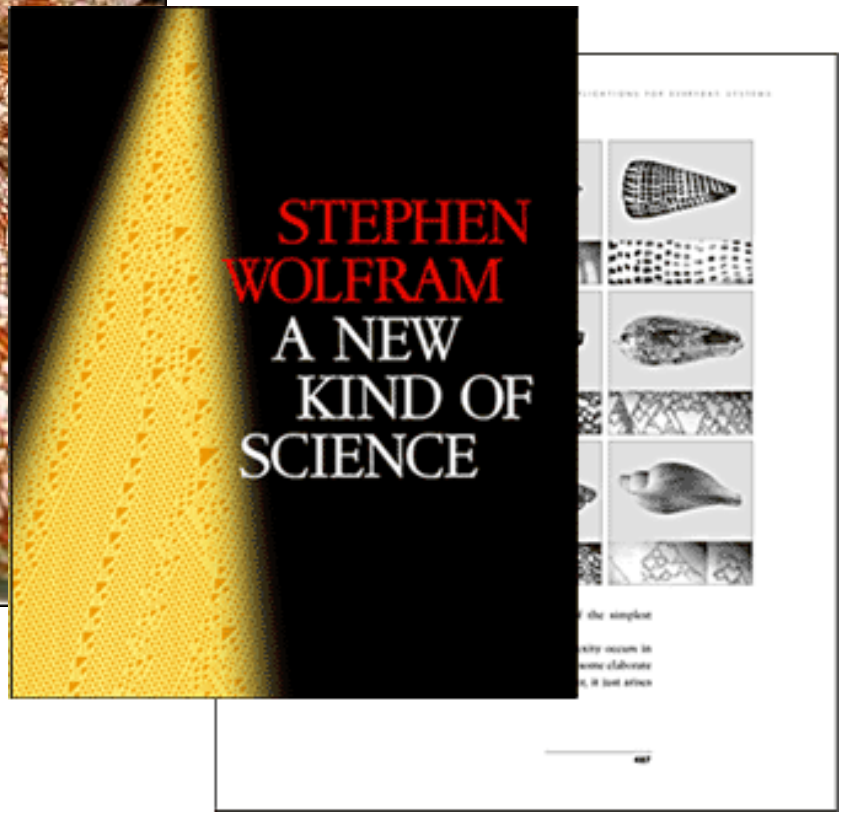
Yet another vision

- Other class of approaches – regressing back to simpler levels
  - Kari Enqvist: "Cognition can be explained in terms of elementary particles"
  - Roger Penrose: "Intelligence + free will are quantum-level phenomena"
  - Stephen Wolfram: "Cellular automata can substitute explicit formulas"



- Of course, phenomena ARE implemented by low-level agents

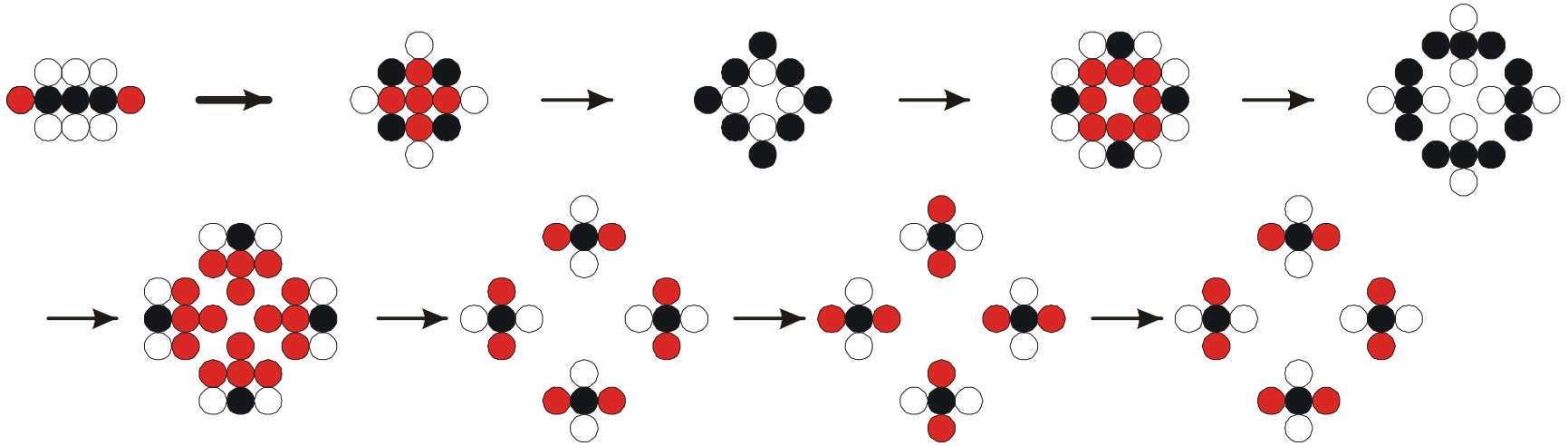
The Ultimate Theory?



- Hot in 2002!
  Not in 2009?

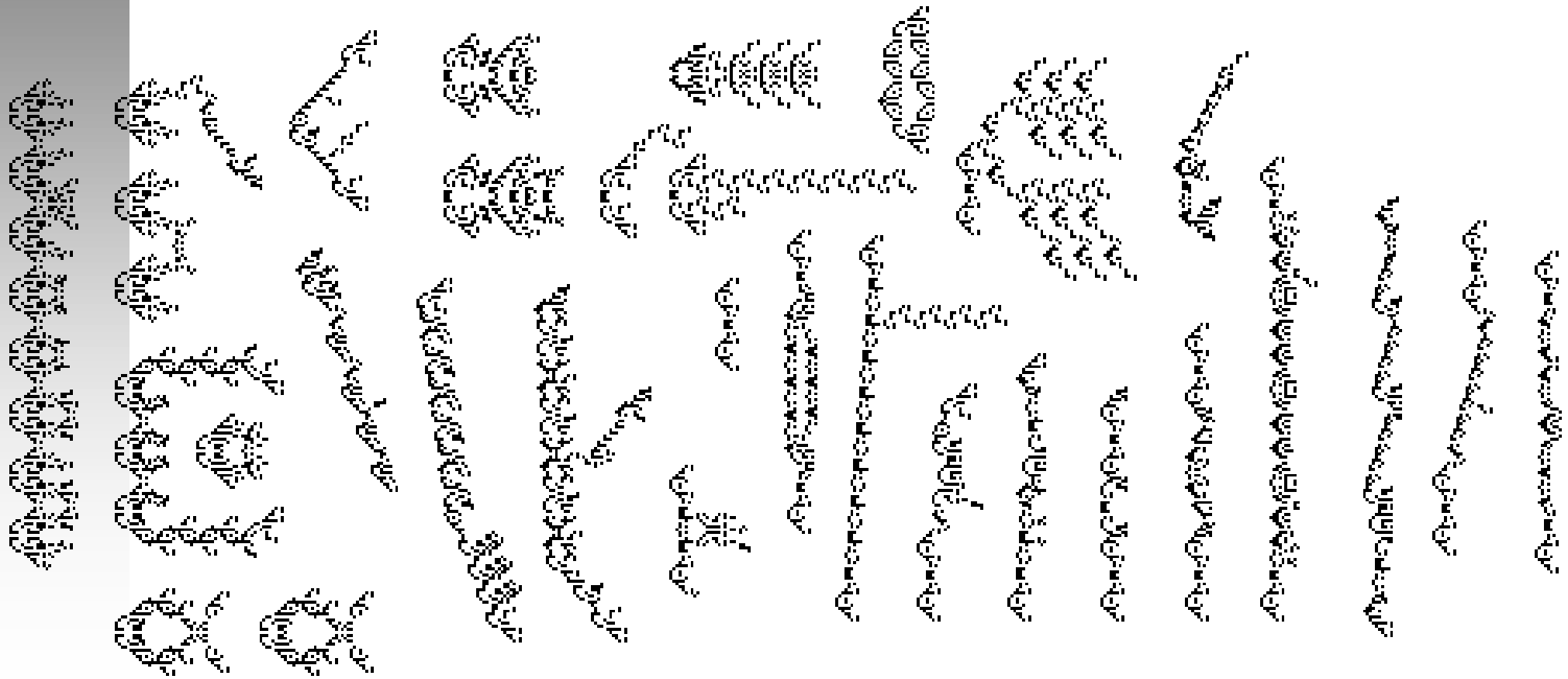
... *Too much* power!

- Wolfram's starting point: Cellular Automata models ...
- ... resulting in a "universal machine" – being unanalyzable!
- Perhaps the simplest interpretation is that the selected model structure is too strong, but Wolfram concludes that ...
- ... this is not only a new theory but a New Science!



John Conway's "Game of Life"

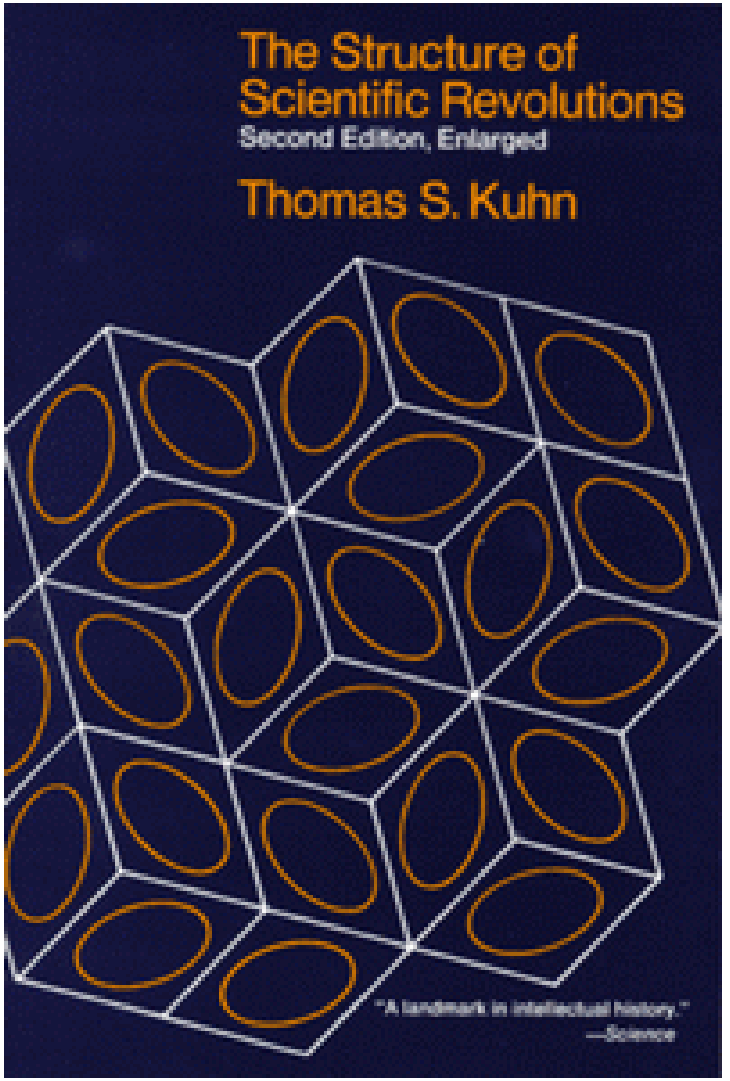
Patterns in the “Game of Life”



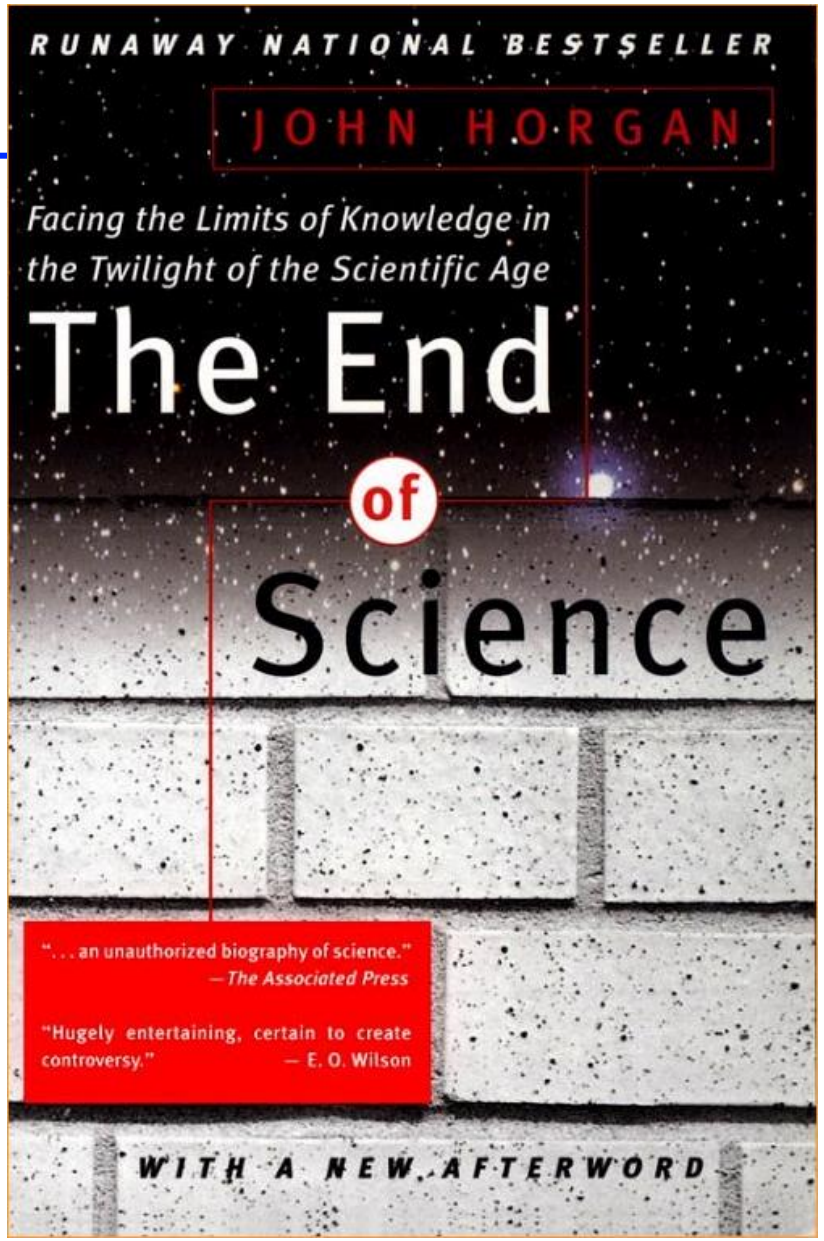
“A New Kind of Science” ...?

- Science on science (Kuhn): *Normal science* fills in the holes in theories
- A *revolution* takes place when the *antitheses* against the old theory cumulate, and a *synthesis* is found, resulting in a *new paradigm*

- All paradigm shifts this far have happened within the framework of “old science”
- What would the “new science” *mean* in the first place?



- Chaoplexity – "ironic science":
  - Unsubstantiated promises
  - Buzzwords, fashions, gurus, ...
- Fuzz around the hot topics has affected traditional schools too
- Applies also to "hard" sciences
  - Physics becoming *metaphysics*
  - Cosmology being based on wild hypotheses (wormholes, multiverses, etc.)
- Counterattack of "old science" – cybernetic turmoil taking place today!

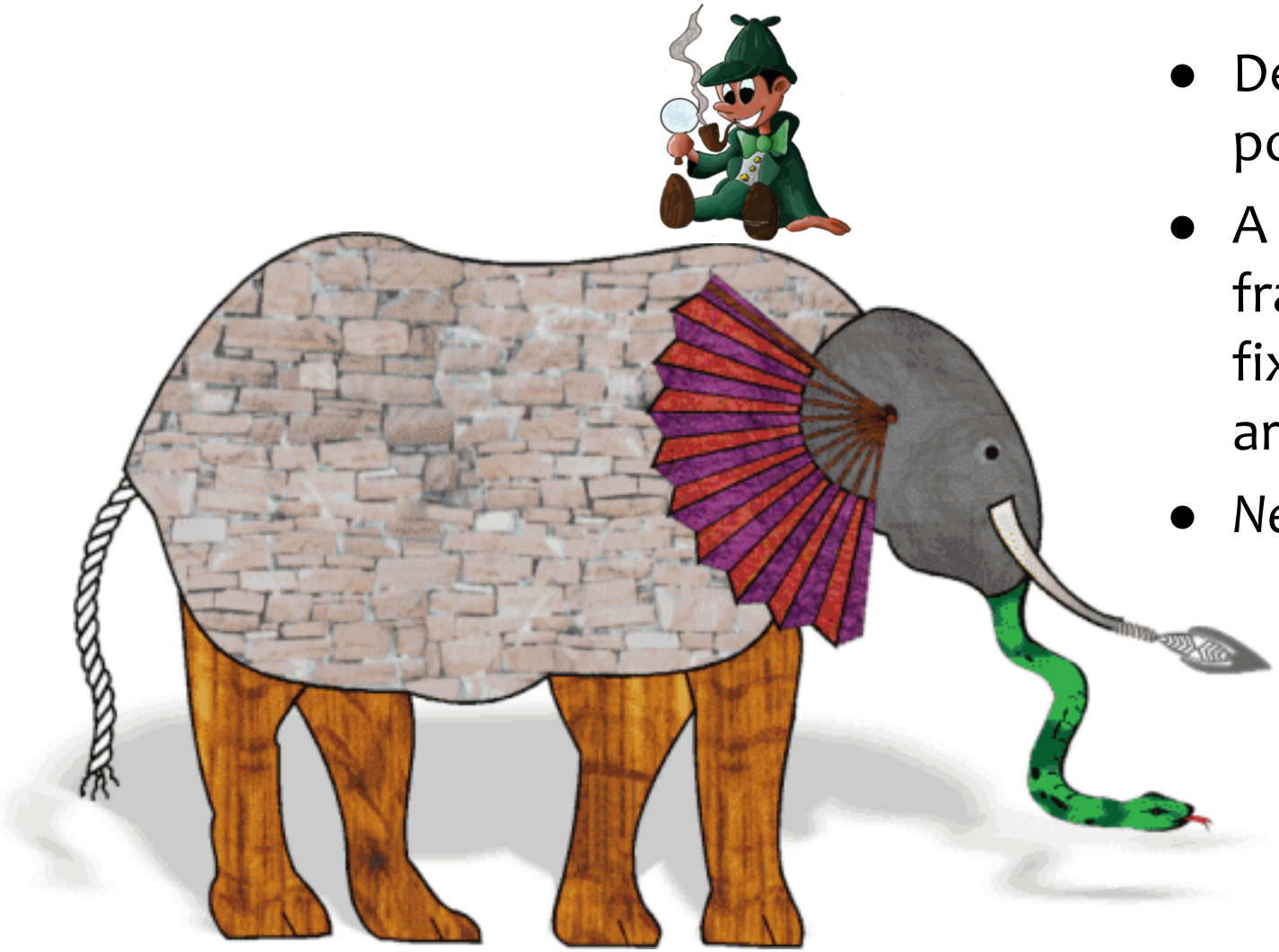


About intuition

- Richard Feynman: You must not try to understand world, "You just have to trust formulas!"
- However, here, when studying systems in general, contrary to Feynman, it is assumed that *intuition is a resource*
- Modeling is about putting one's understanding into concise (mathematical) form
- Now: Try to stay on the "edge of chaos" between scientific method and chaoplexity intuitions
- When facing complex systems, intuition is the *only* resource there is when trying to capture the true essence

Problem: Everybody has his/her *own* intuition

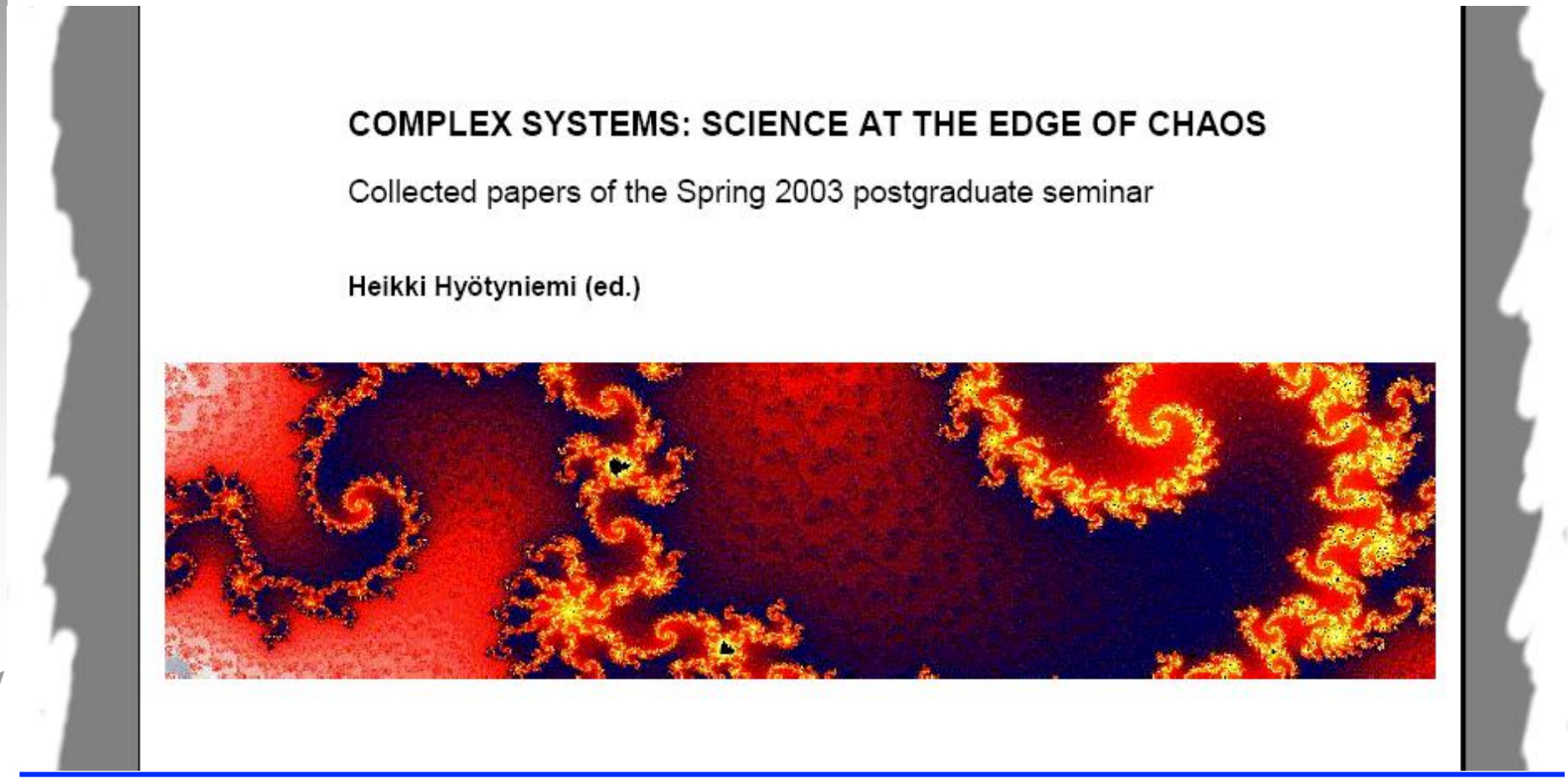
... What is this system?



- Depends on the point of view
- A consistent framework and fixed concepts are needed
- *Next lesson ...*

Report 145

- More material on the topics in complex systems research



http://neocybernetics.com/report145/

Chaos Concluded

- There are some lasting results reached in chaos theory.
- Perhaps one of them is the *universality of bifurcation behavior* (as studied by M. Feigenbaum)
- Another such result is surely *Sharkovskii's theorem*:

Suppose that f is a real-valued continuous function. We are interested in the possible periods of f. Consider the following ordering of the positive integers:

$$3, 5, 7, 9, \ldots, 2\cdot3, 2\cdot5, 2\cdot7, \ldots, 2^2\cdot3, 2^2\cdot5, \ldots, 2^4, 2^3, 2^2, 2, 1.$$

That is, start with the odd numbers in increasing order, then 2 times the odds, 4 times the odds, etc., and at the end put the powers of two in decreasing order.

Sarkovskii's theorem states that if f has a periodic point of period m and $m \leq n$ in the above ordering, then f has also a periodic point of period n.

- This fact implies the famous observation that "period three implies chaos".

Example

- Define the continuous mapping f as

$$x(k+1) = f(x(k)) = \begin{cases} 1 + x(k), & \text{when } x < 0 \\ 1 - 2x(k), & \text{when } x \geq 0 \end{cases}$$

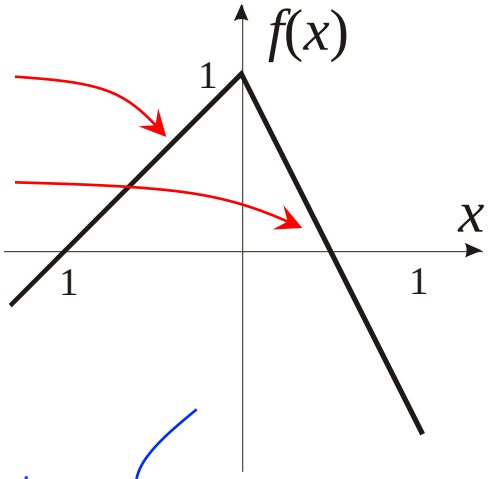
- This mapping has period three because

$$f\left(f\left(f(1)\right)\right) = f\left(f(-1)\right) = f(0) = 1$$

  Thus, it must have a period of arbitrary length!

- Is it possible to determine such cycles in practice? – In this case, *it is indeed possible*, as shown below.

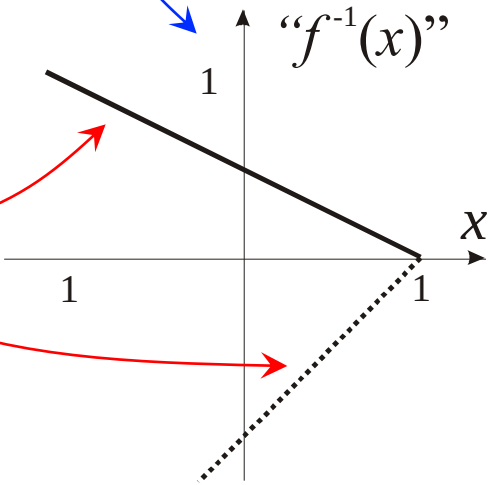
$$x(k+1)=\begin{cases} f_1(x(k))=1+x(k), & \text{when } x<0 \\ f_2(x(k))=1-2x(k), & \text{when } x\geq 0 \end{cases}$$



- One is only interested of the periodicity properties here – this means that the same cycle can be studied stepping in the "inverse direction":

$$x(k-1)=\begin{cases} f_1^{-1}(x(k))=x(k)-1 \\ f_2^{-1}(x(k))=\frac{1}{2}\left(1-x(k)\right) \end{cases} \quad or$$

Invert axes



Here, *either* of the branches can be selected at a time.

- Above, the constraint $x(k) < 1$ remains always valid if the branch 1 is selected only once in succession.

- For example, a four-step cycle can be found as

$$\begin{aligned} x(k) &= f_2^{-1}\Big(f_2^{-1}\Big(f_2^{-1}\Big(f_1^{-1}(x(k))\Big)\Big)\Big) \\ &= \tfrac{1}{2}\Big(1-\tfrac{1}{2}\Big(1-\tfrac{1}{2}\big(1-x(k)+1\big)\Big)\Big) \\ &= \tfrac{1}{2}-\tfrac{1}{8}x(k) \end{aligned}$$

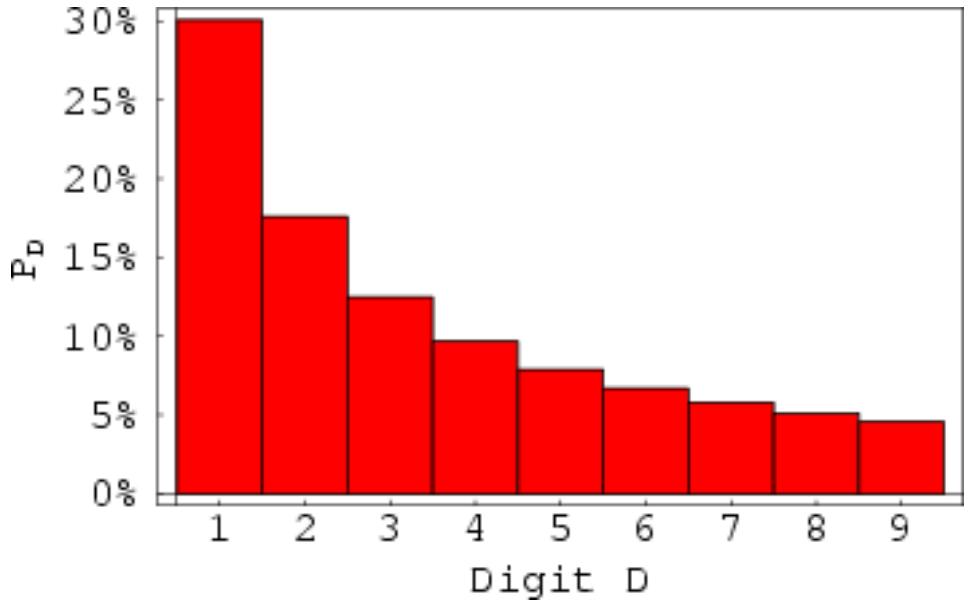
  giving the solution $x(k) = 4/9$. *Test it!*

- Because of piecewise linearity, any cycle length analyzable!

Bonus: "Benford law"

- Result of scale invariance:

  The first digit in a real-life number is 1 more probably than some other



Fundamental Physical Constants

- Version of this page using math mode *(you need a browser such as Arena!)*
- PostScript Version

(without warranty)

Planck constant h

$6.6260755 \cdot 10^{-34}$ J·s

$h / (2\pi) = 1.0545726 \cdot 10^{-34}$ J·s

Boltzmann constant k_B

$1.38065 \cdot 10^{-23}$ J/K $\quad (= 8.617385 \cdot 10^{-5}$ eV/K)

Elementary charge e

$1.602177 \cdot 10^{-19}$ C

Avogadro number N_A

$6.0221367 \cdot 10^{23}$ particles/mol

Speed of light c

$2.99792458 \cdot 10^{8}$ m/s

Permeability of vacuum μ_0

$\mu_0 = 4\pi \cdot 10^{-7}$ T^2·m^3/J

$12.566370614 \cdot 10^{-7}$ T^2·m^3/J

Permittivity of vacuum ε_0

$\varepsilon_0 = 1 / (\mu_0 c^2)$

$8.854187817 \cdot 10^{-12}$ C^2/J·m

Fine structure constant α

1 / 137.0359895

Electron rest mass m_e

$9.1093897 \cdot 10^{-31}$ kg

Proton rest mass m_p

$1.6726231 \cdot 10^{-27}$ kg

Neutron rest mass m_n

$1.6749286 \cdot 10^{-27}$ kg

Bohr magneton μ_B

$\mu_B = e h / (4\pi m_e)$

$9.2740154 \cdot 10^{-24}$ J/T

Nuclear magneton μ_N

$\mu_N = e h / (4\pi m_p)$

$5.0507866 \cdot 10^{-27}$ J/T

Free electron g factor g_e

2.002319304386