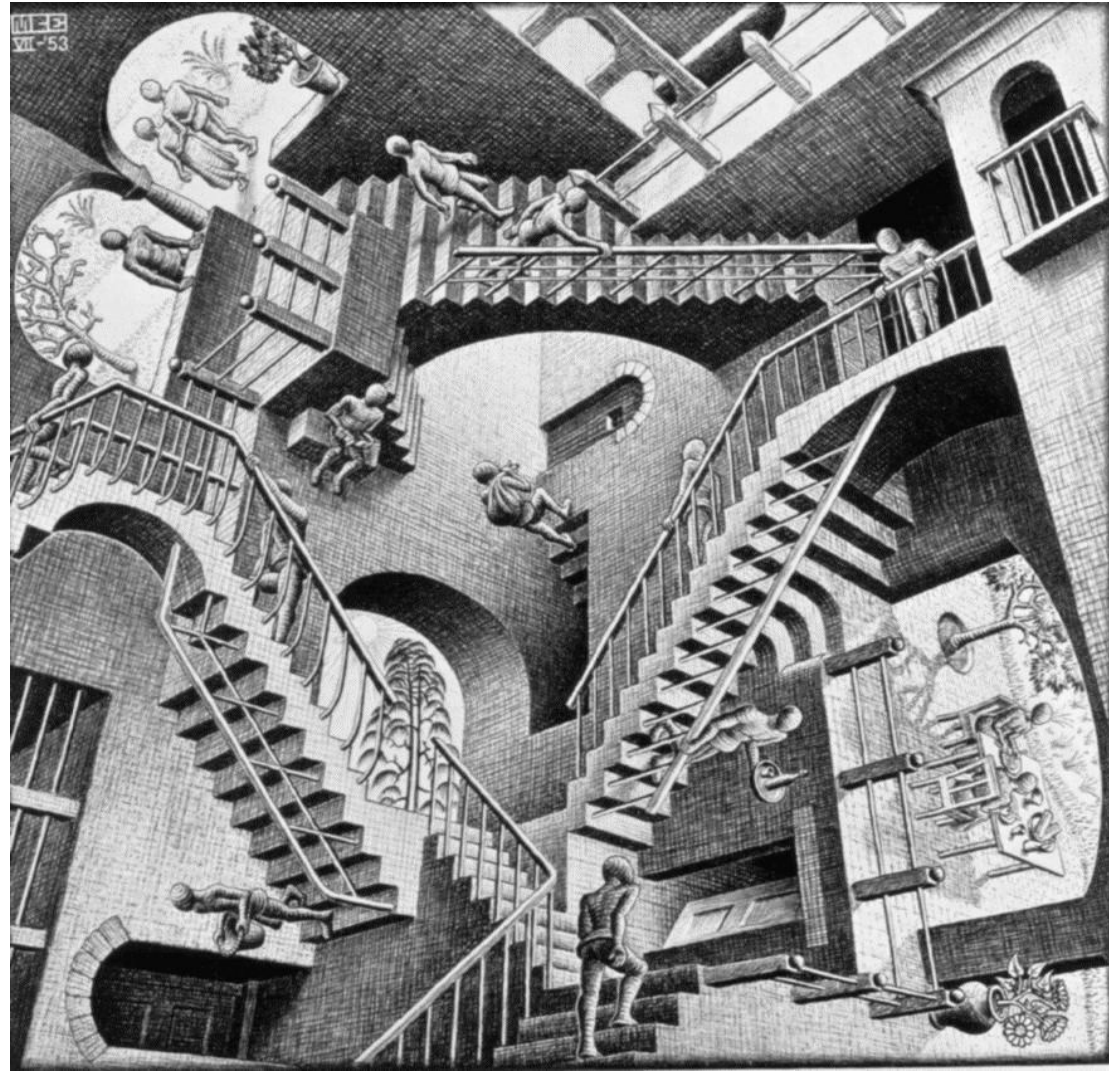

AS-74.4192 Elementary Cybernetics

Lecture 3:
**Towards Modeling
of Emergence**



Where to go?

- First: mindstorming, apply the intuitions!
- Then: select the directions where one can proceed
- Vision + mission: Holistic problems, but reductionistic methods
- First: “Model of modeling”



Typical evolution of models: “Bottom up”

- Models tend to become more and more sophisticated
- For example, in bioprocess modeling, using the basic bricks

Exponential

$$\frac{dx}{dt} = \mu x$$

Logistic

$$\frac{dx}{dt} = \mu x - kx^2$$


Monod model

$$\frac{dx}{dt} = \frac{\eta s}{k + s} \cdot x$$

one can construct:

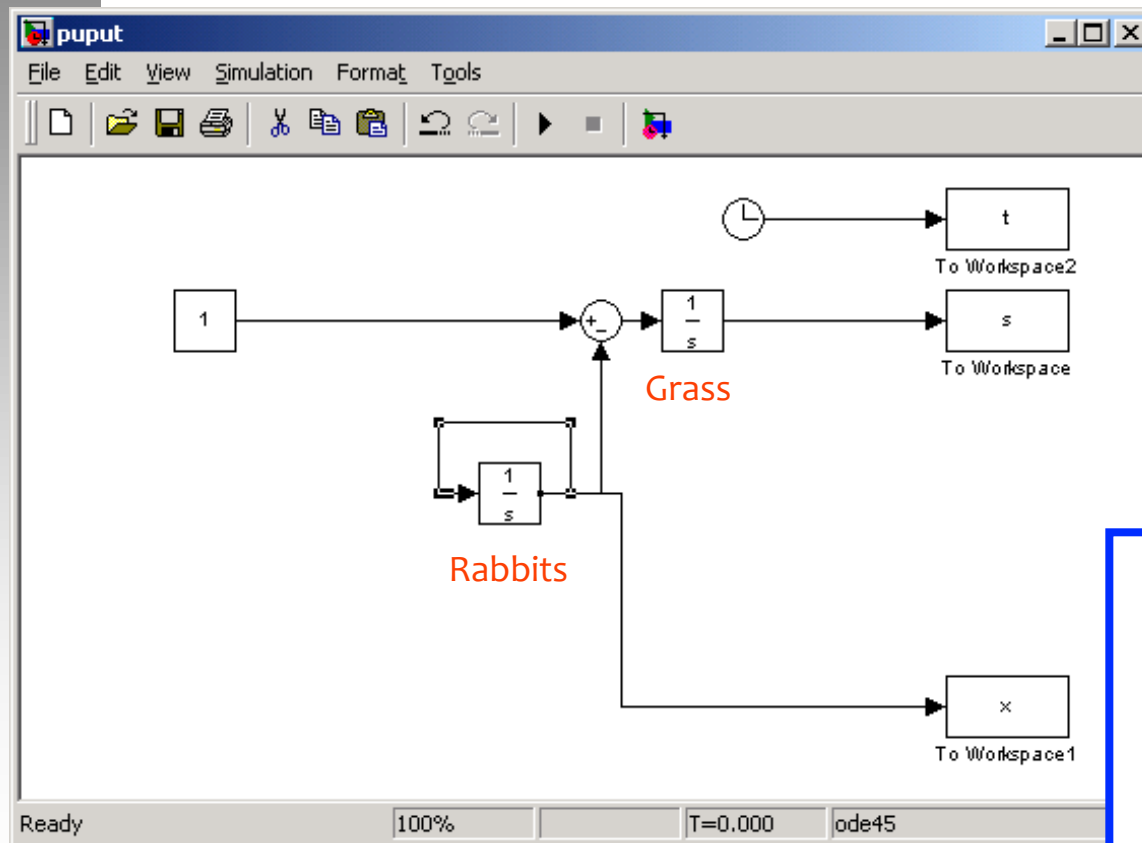
*Fiddler's
paradise!*

Models
unanalyzable &
hardly useful


$$\left\{ \begin{array}{ll} \frac{dx}{dt} = \mu x - k_1 x b & \text{Biomass} \\ \frac{ds}{dt} = -k_2 s x - k_3 \frac{s}{k_4 + s} x & \text{Substrate} \\ \frac{da}{dt} = k_5 s \frac{1}{k_6 + b} x - k_7 \frac{a}{k_8 + a} x & \text{Acid} \\ \frac{db}{dt} = k_9 s x - k_{10} \left(k_5 s \frac{1}{k_6 + b} x - k_7 \frac{a}{k_8 + a} x \right) & \text{Alcohol} \end{array} \right.$$

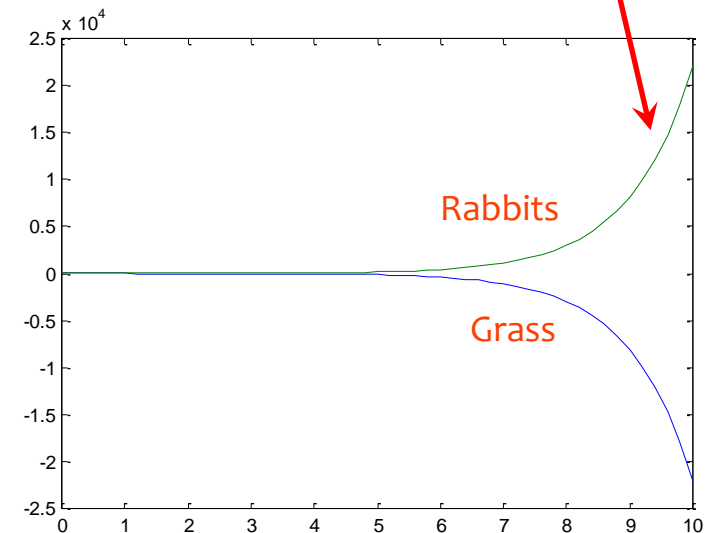


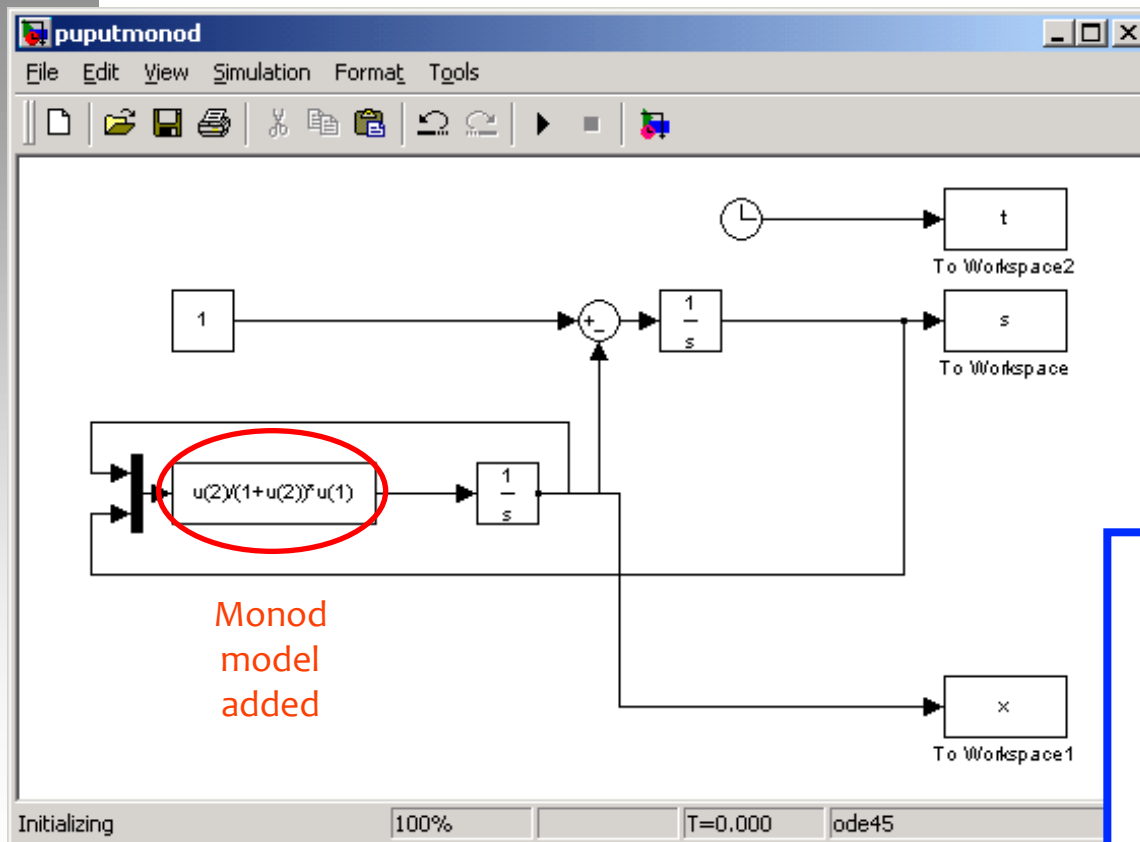
Example: Model of Grass vs. Rabbits



First try

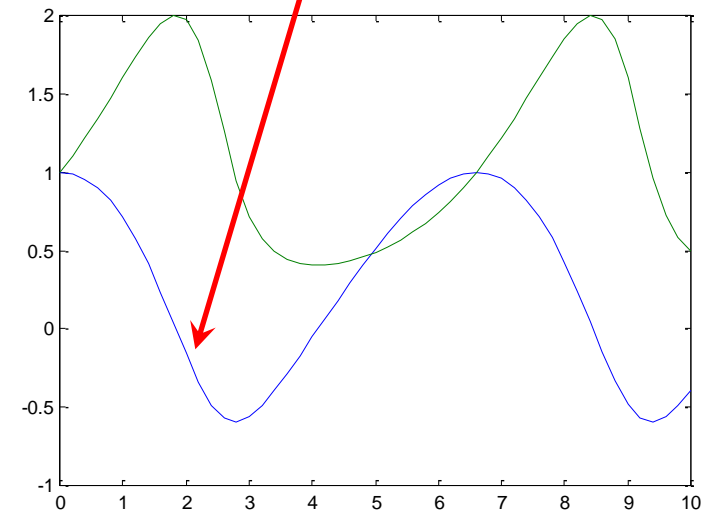
- Needs fixing: unlimited growth

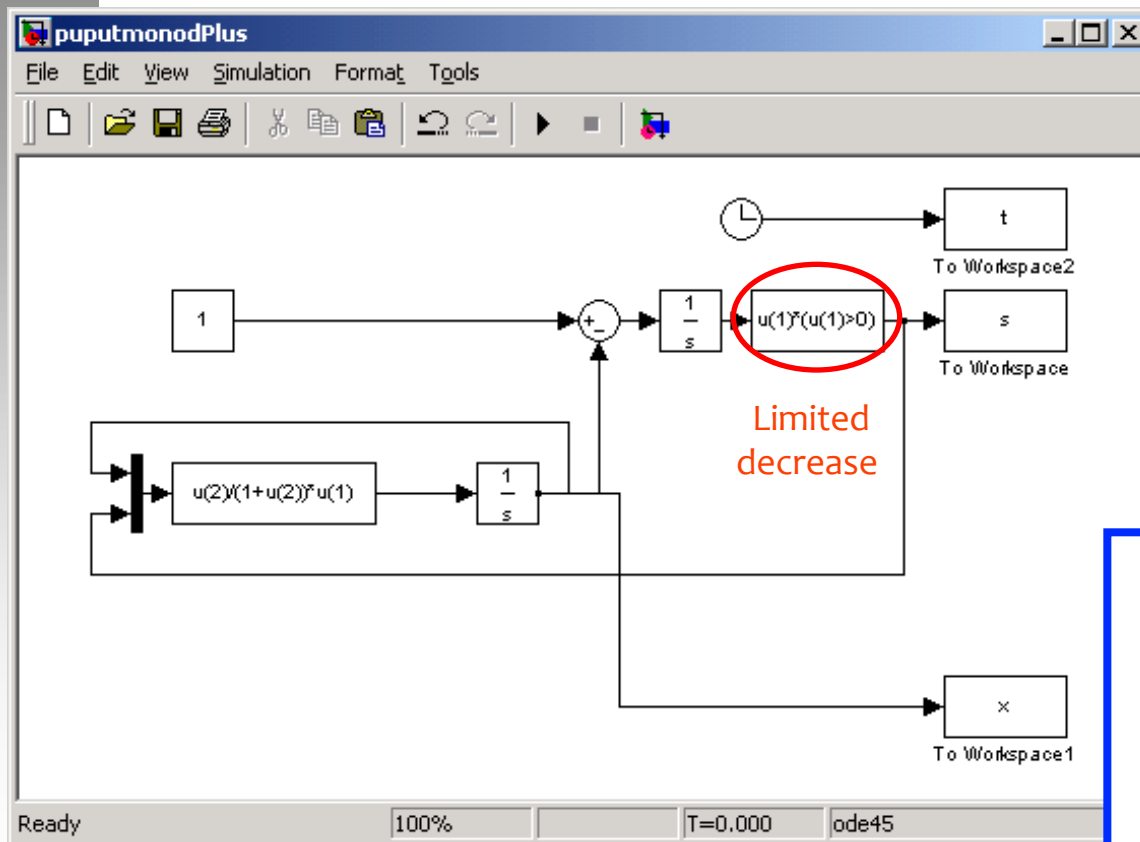




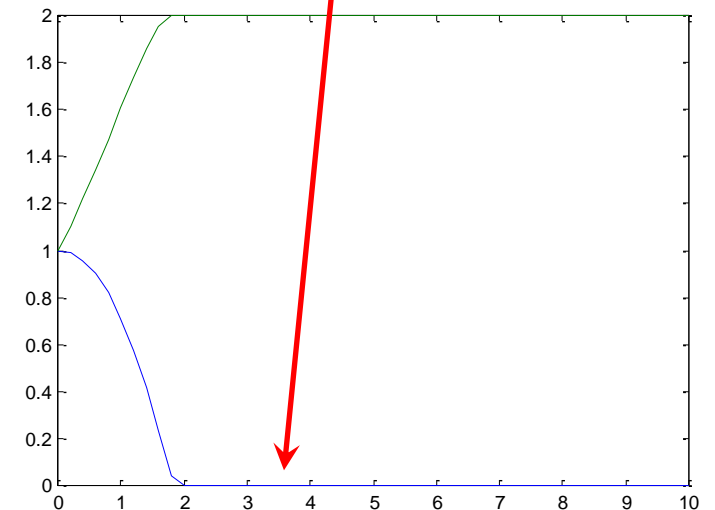
Still needs fixing ...

- Negative biomass!

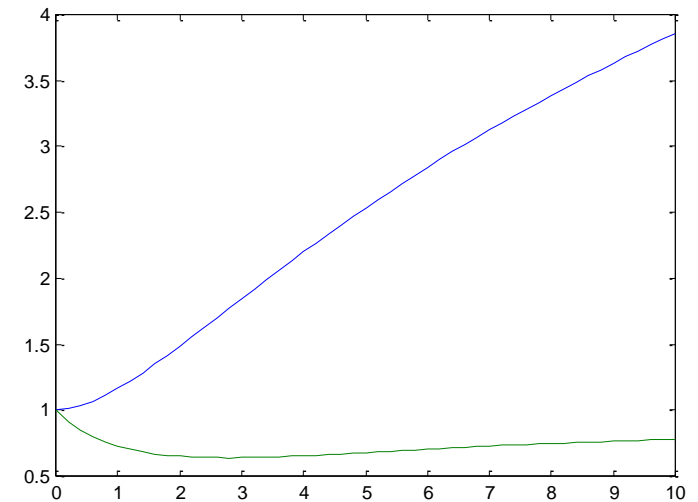
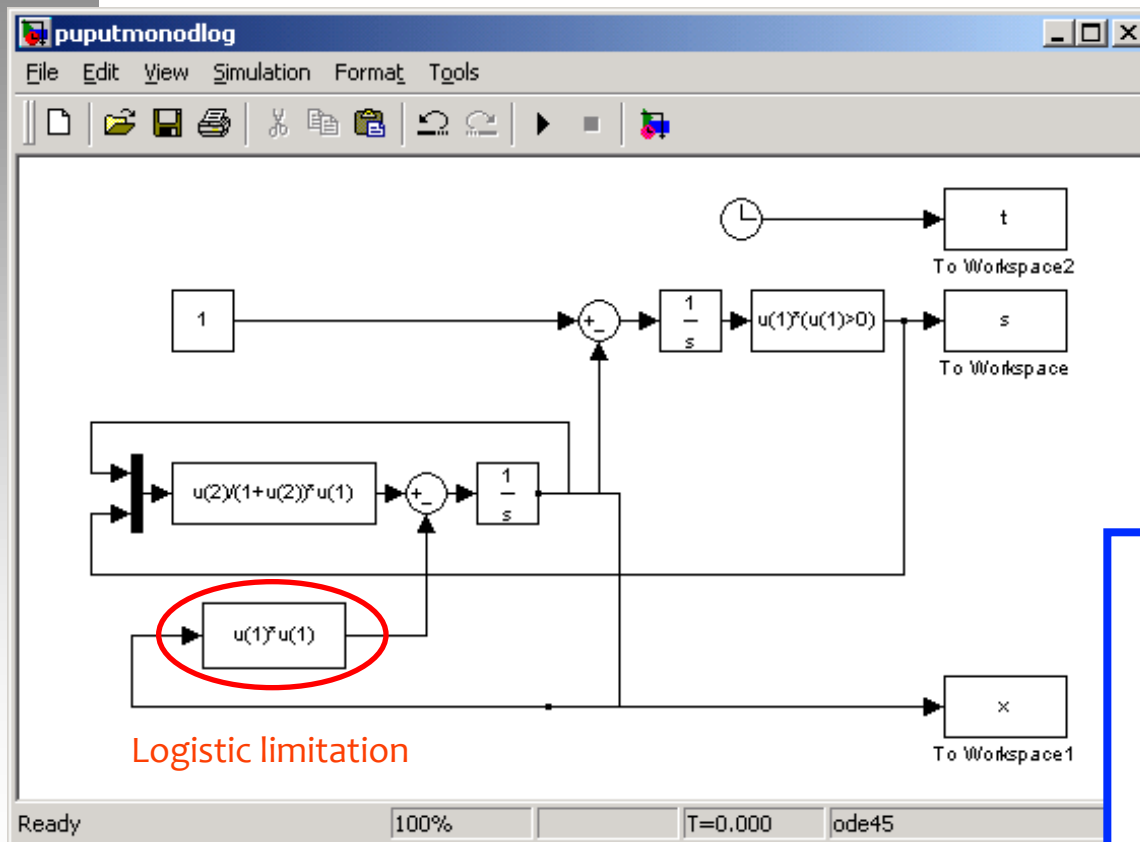




... and still ...



... An endless task!

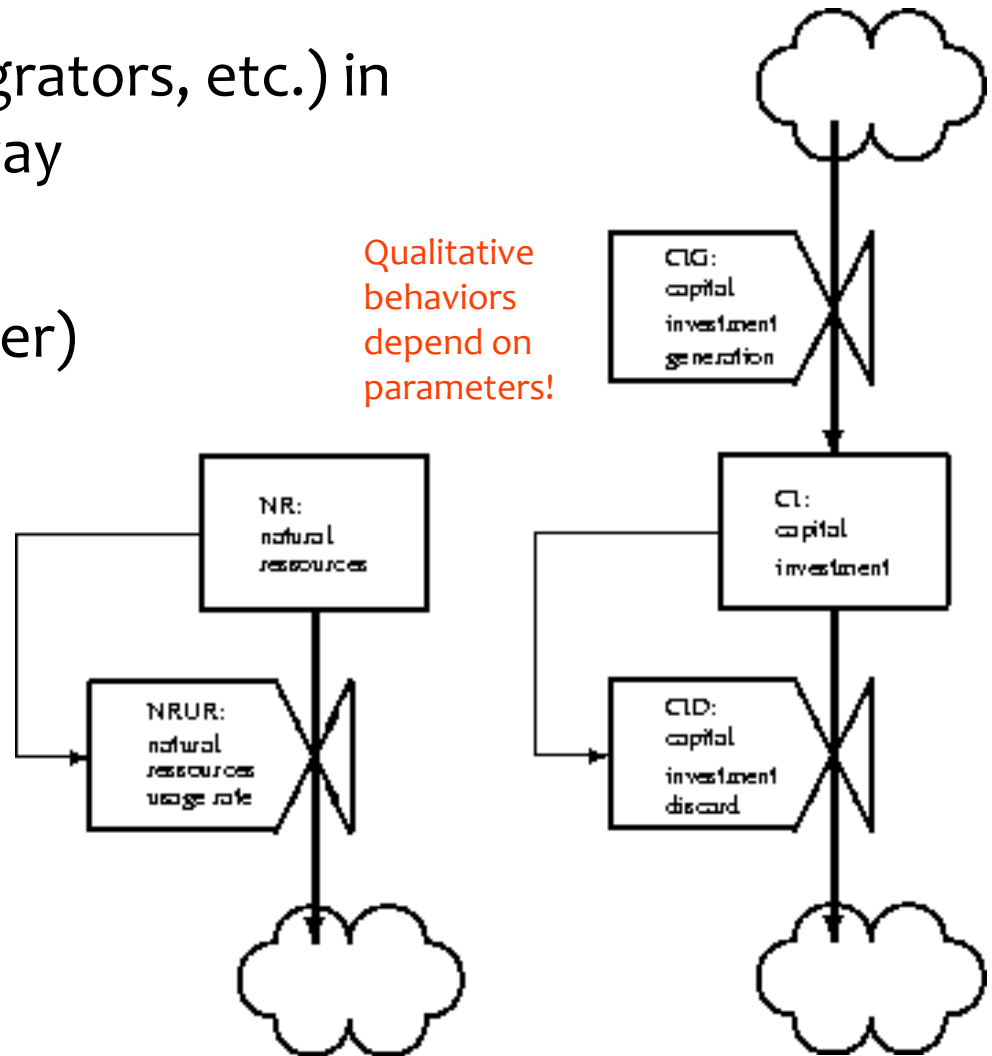


Similarly in all complex environments ...

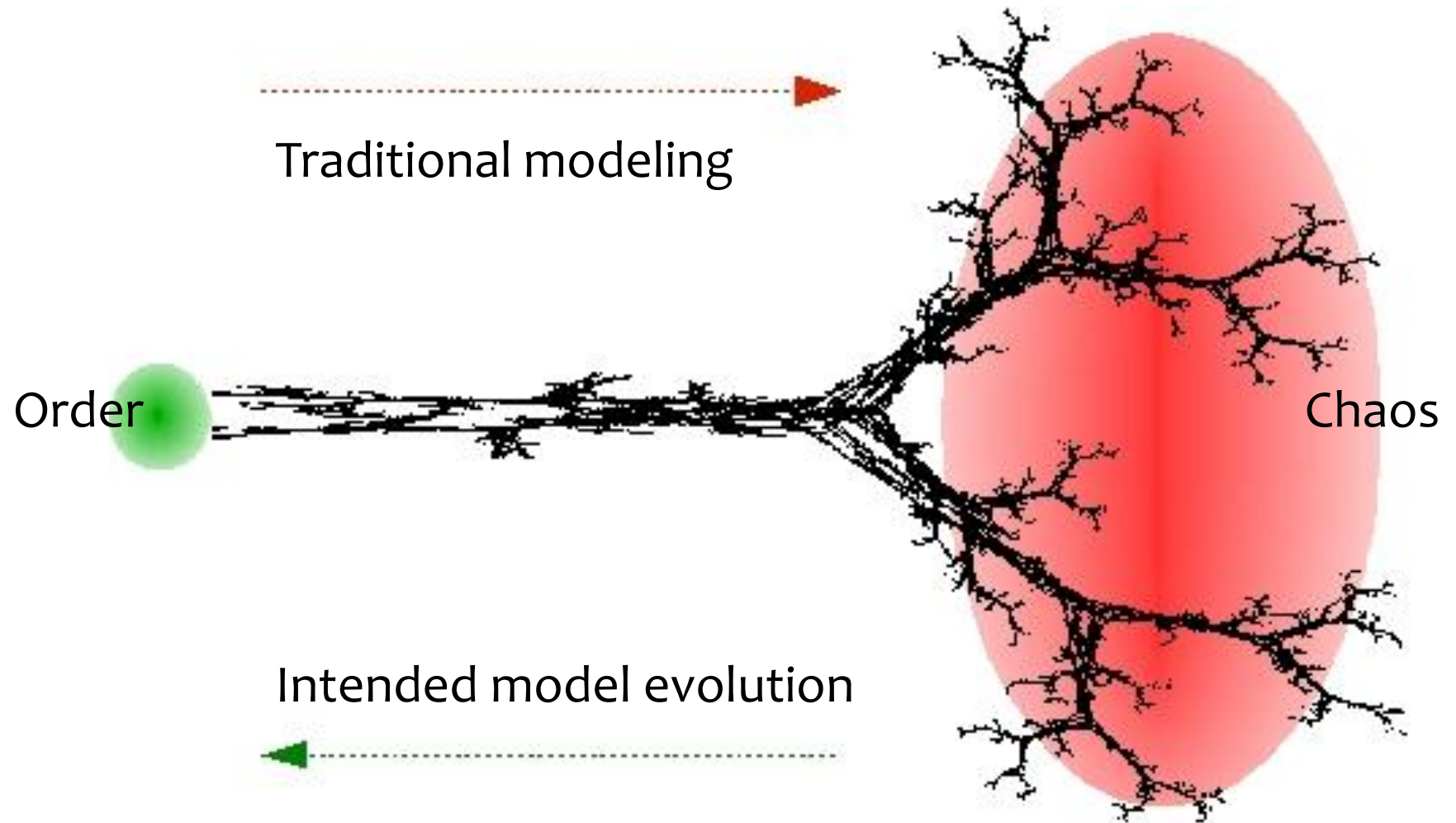
- Combine basic blocks (integrators, etc.) in an intuitively appropriate way
- Example: *System dynamics*
- World models, etc. (Forrester)
- However:

Simulation exactly reveals what was explicitly modeled!

Are there other alternatives for finding models?



Are there alternative approaches?



“Top down”: Power of systemic thinking

Assume that the door of a refrigerator (1 kW, $\eta = 30\%$) is left open. How will the room temperature behave?

1. It will decrease.
2. It will increase.

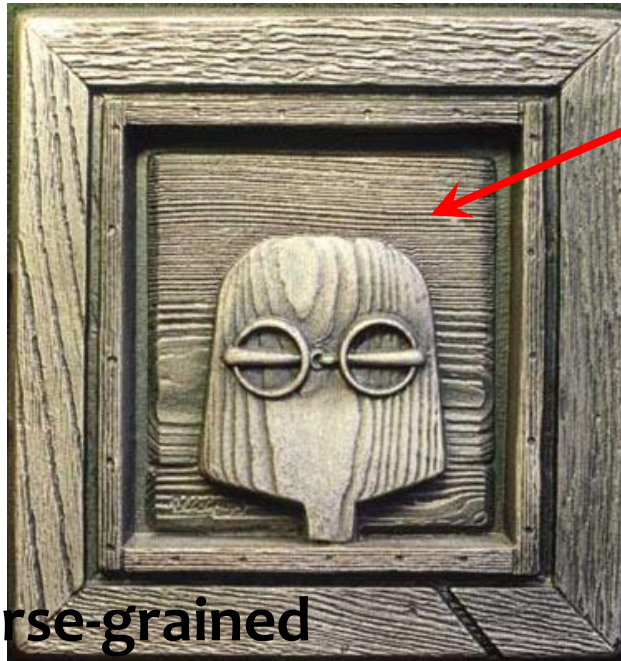
– But *what is the power?*

- Appropriately defining the system boundaries makes this simple!



“Granularization”

- Fewer number of variables selected to represent a phenomenon
- *Generalize & abstract away!*



coarse-grained



fine-grained



Warning

- Ideal of holism: The whole is larger than the sum of the parts
- Risk of holism: The “hole” is larger than the sum of the parts:



The temptation of loose hypothesizing has to be avoided, abstractions must not be too wild



Key question – link among levels = *emergence*

1. What do you think is an emergent phenomenon?
2. How would you approach and model it?



Complex systems: Science in the making

- The field of complex systems research is far from mature
- No paradigmatic guidelines yet exist: There are no generally approved approaches, common concepts, methodologies or tools, typical application domains or problem settings
- **Neocybernetics** is an approach to capturing the essence of complexity in a simple framework
- **Now:** The concepts and underlying assumptions are defined
- **Note:** The developments were not originally so straightforward – only “highway through the jungle” is shown
- **Note:** There always exist many ways where to proceed; here, alternative branches also need to be considered ...



Opposite intuitions #1

- Traditionally, in complex systems research emphasis is on *surface patterns*, visible formations
 - For example, Wolfram's sea shells
 - Fractal intuitions, etc.
- However, remember Alan Turing:
“The zebra stripes are simple – I am more concerned of the *horse* behind”
Compare to shallow vs. deep views to AI
- **Now:** key point in complex systems is not the surface patterns but *deep structures*

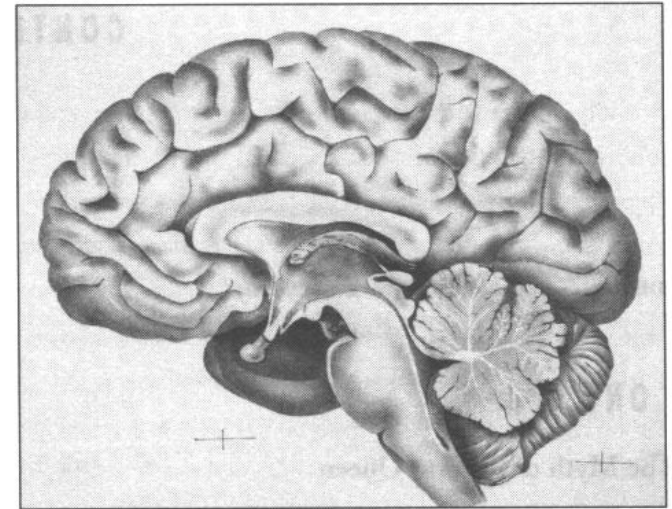


Diagram of the human brain (Courtesy of Mittermeier)



Map of Hamburg, circa 1850 (Courtesy of Princeton Architectural Press)

from S. Johnson: “Emergence”

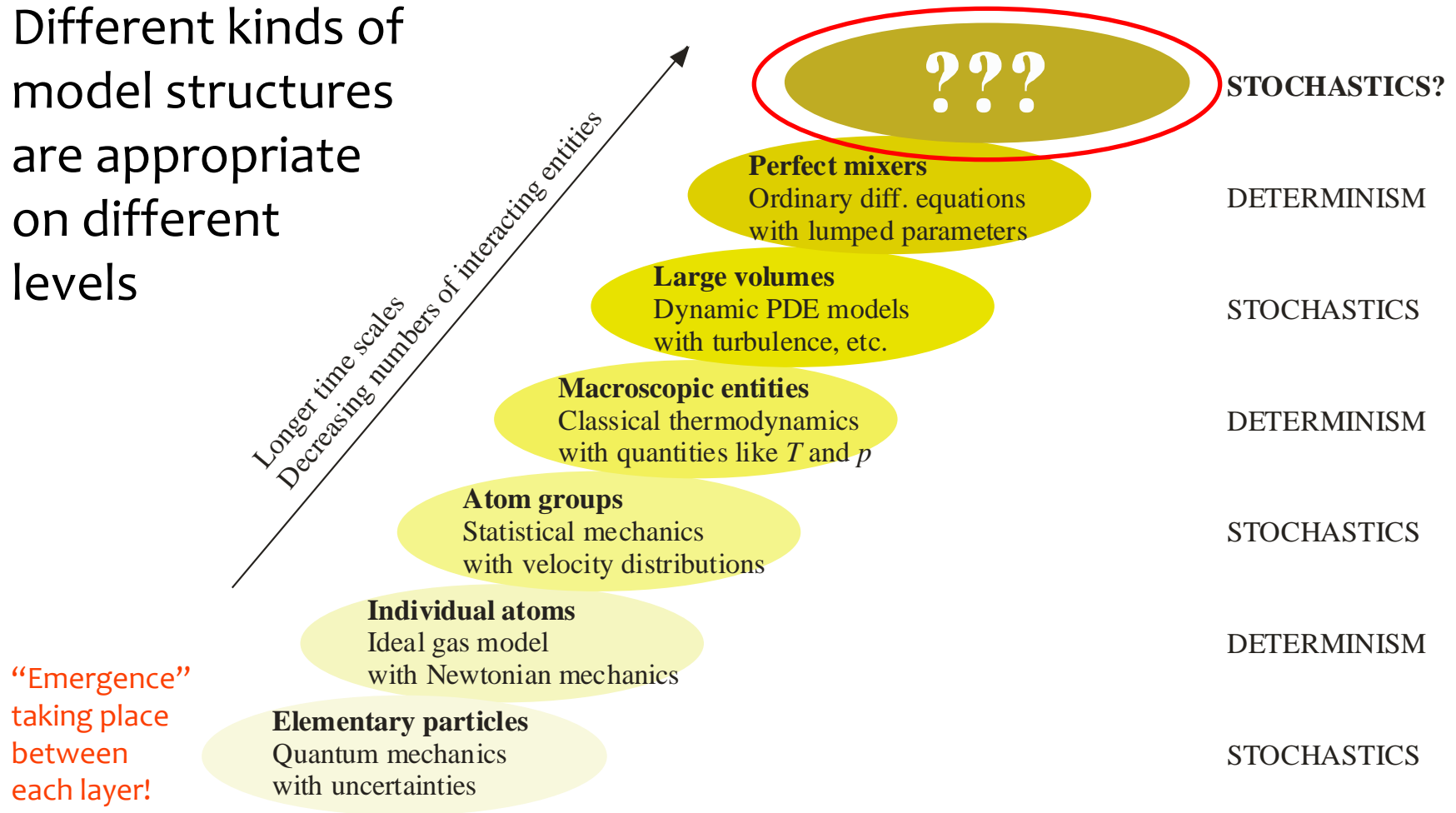


-
- Assume: *Deep structures = underlying emergent patterns* meaning *functionalities* where the system is attracted to
 - How to model emergence (a *holistic* phenomenon) in a reductionistic way ...?!
 - Indeed, apply the very traditional modeling ideas:
 - First: Studying explicit examples, construct an intuitive understanding of what emergence is
 - After that: Find the common features and represent them in an explicit mathematical framework
 - Sounds like nonsense? However, this *can be done*.
- ... There are many ways to interpret the evidence ...



Emergence? – Seen it before!

- Different kinds of model structures are appropriate on different levels



- Today, the level of deterministic first-principles models is already fully exploited
- Now one should reach for the highest, stochastic level
(Note: Stochastic and deterministic levels alternating is no coincidence: Otherwise the levels could be “collapsed”)
- Higher level: *Time scales longer + number of functional entities lower* (granularizing lots of faster parallel interactions)
- Abstractions in emergent models:
 - Time axis has to be eliminated!
 - Individual realizations have to be ignored!
- This must be done in an appropriate way, so that the properties relevant on the higher level are not compromised



Towards formalizing the intuitions

- ... You have one grain of sand, then you have two, then three – at what point do you have a *pile* of sand?
- Two concepts that are intuitively close to each other: **emergence** and **infinity**
- Abstracting details away along the time axis or spatial axis:

$$E\{quantity\} \stackrel{\text{def}}{=} \lim_{t \rightarrow \infty} \left\{ \frac{1}{t} \int_{-t}^0 quantity(\tau) d\tau \right\}$$

- Here, *expectation* “E” can be interpreted as “emergence”
- Individual instances have no relevance whatsoever any more; time-domain behaviors become represented by end results



-
- **Is emergence only averaging?!**
 - Appropriate nonlinearity is here needed – then, *effective theories* can be found (standard view)
 - For example, *gas temperature* follows weak emergence:

$$T \sim \bar{E}_{\text{kin}} \sim \text{E}\{v^2\}$$

- What is needed for (more intuitively appealing) emergence is *interaction of underlying components* (new view!)
- In its simplest form, such *coupling* can be modeled as

$$\text{E}\{x_i x_j\}$$

Note that covariances can still be modeled in linear terms!

- This all is captured in *covariances* (next time!)



NEW

- Traditional computability theory has only studied cases where algorithms stop – in the Greek spirit truly!

The Halting Problem



NOT
THAT'S THE QUESTION!

- Such theory cannot study emergence because all computations with E are *infinite* (at least in principle)
- In practice, relevant data typically behaves nicely, iterations converging towards final values also for finite sample sizes
- Rather than studying formal algorithms or other structures, emphasis and starting point now is on data properties!



Opposite intuitions #2

- Traditionally, one concentrates on individual actors and distinct time points
- Now one abstracts *from individuals to groups or populations, from distinct anomalies to averages or expectations*
- Consequences – some examples:
 - The “selfish gene” and game theoretic individualistic approaches cannot explain *altruism*, etc., that become reasonable only at the population level
 - “Survival of the fittest” is not the key point – matching the whole terrain, finding the outlook of “fitness landscape” is the goal of a population
 - Family of good solutions and modeling their spectrum – this is assumedly what nature also does: It does not know how to solve NP hard problems!
- Disadvantage: The resulting models cannot then simulate individual actors or processes



- Paradoxes are like “super colliders” of traditional thinking: problems and new horizons are momentarily exposed
 - For example, study the “prisoner’s dilemma”. There are two prisoners, A and B, who are accused of the same crime. If they co-operate and do not deceive each other, the penalty is not very bad ... on the other hand, if either one deceives the other, he will walk free, while the other will have the maximum penalty – long sentences are given also if both deceive.
 - Traditional game theory says that one should always deceive.
 - Co-operation emerges as a winning strategy only in “iterated prisoner’s dilemma” case, where – again – the time axis is eliminated.

Payoff to player A

		Player B	
		C	D
Player A	C	2	0
	D	3	1



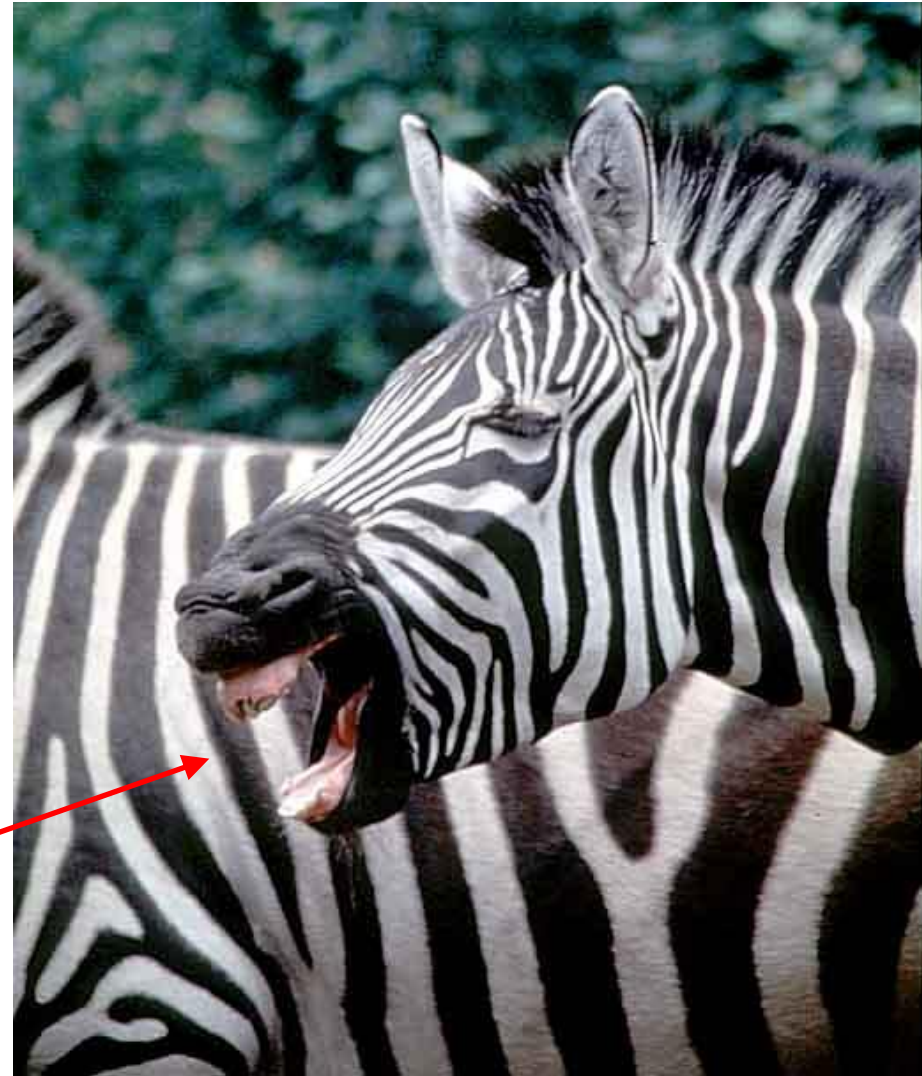
Opposite intuitions #3

- Phenomena can be represented in terms of *processes* or in terms of *patterns* (Herbert Simon 1969)
- When describing complex systems, process view dominates:
 - Individual (inter)actions and explicit time structure is emphasized; no doubt because such causal structures are easier to grasp
 - For example, traditional cybernetic models are based on actual realizations of sequential, more or less one-at-a-time interactions
 - Thinking in terms of programs; for example, the *agents* today are software constructs
 - And all AI techniques are today seen in such agent perspective (for example, Russell, Norvig: “Artificial Intelligence: A Modern Approach” states that “... The unifying theme of the book is the concept of an *intelligent agent*. ...”)
- **Now:** *Patterns* are determined using *statistical properties*
Counterintuitively, one can address *process philosophies*



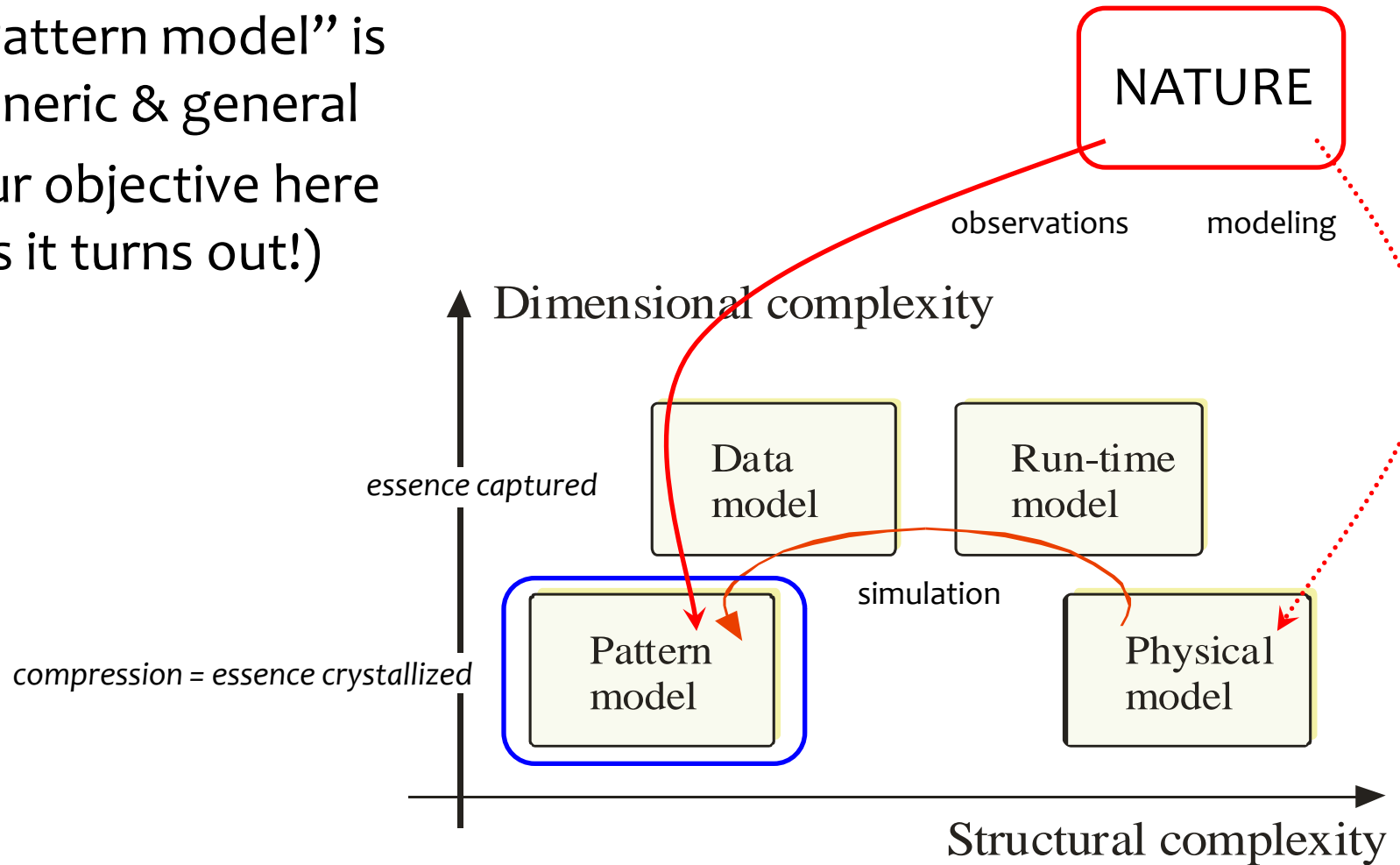
... Process view (cont'd)

- Why process view rules?
Some intuitions:
 1. Because computer is used as a general modeling tool, algorithmic view dominates
 2. Specially, the chaos theory defines iteration as the route to complexity
 3. Also: There is more to the phenotype than there is information in the genotype



Vision now / Technical view

- “Pattern model” is generic & general
- Our objective here (as it turns out!)

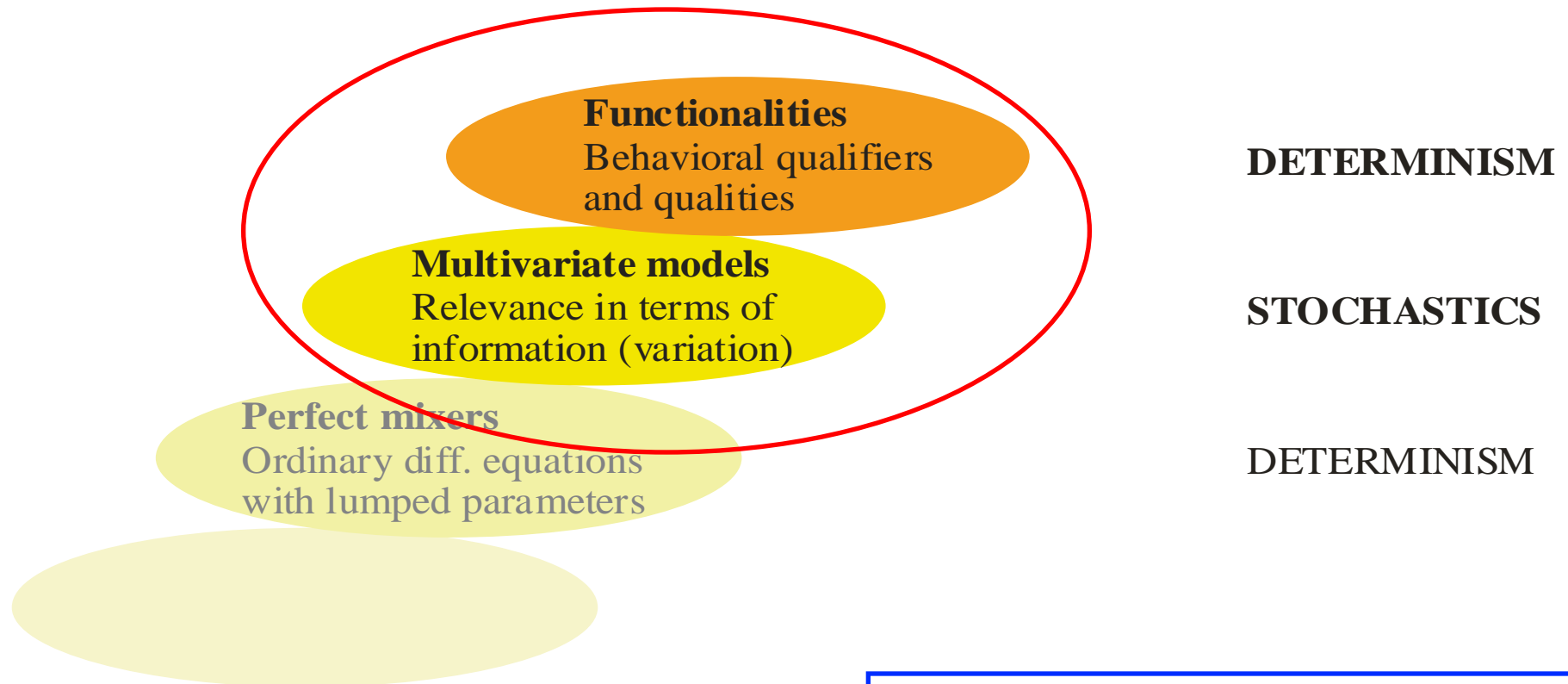


Opposite intuitions #4

- Traditionally, physical model tries to capture one system, one phenomenon in isolation
- Purely syntactical fixed model structure as the starting point – a priori assumptions about the system are needed
- “TRUTH” = system behavior with no disturbances
- Now, pattern model integrates the system and its environment
- System is in interaction with the surroundings; this reveals its role in the environment, giving a “semantical” representation
- RELEVANCE = System with “disturbances”, typical auxiliary effects, resulting in the essence of the system



- New levels in the emergence hierarchy (to be seen later)
- Highest level consists of semantically meaningful patterns



Kantian-style compromise:
Structure of the patterns theory driven
Contents of the patterns data defined!

Importance of *stationarity* and *stability*

- Many problems fade away when the actual dynamic processes are abstracted using statistical system properties (these problems are tackled with in control engineering)
- When can such abstractions be carried out?
 - To have statistical measures emerge, the signals have to be *stationary*
 - To have stationary signals, underlying system has to be *stable* in the large
- However, there are tensions in a system: The cybernetic balance is a *dynamic equilibrium* (in the large)
- Heuristics: Stability constitutes a “cooker” of complexity
- Truly, how could one assume stability in natural processes?
 - The other alternatives are *explosion* (resulting in exhaustion of resources) or *extinction*



-
- Note that there is a big difference between momentary patterns and dynamic equilibria

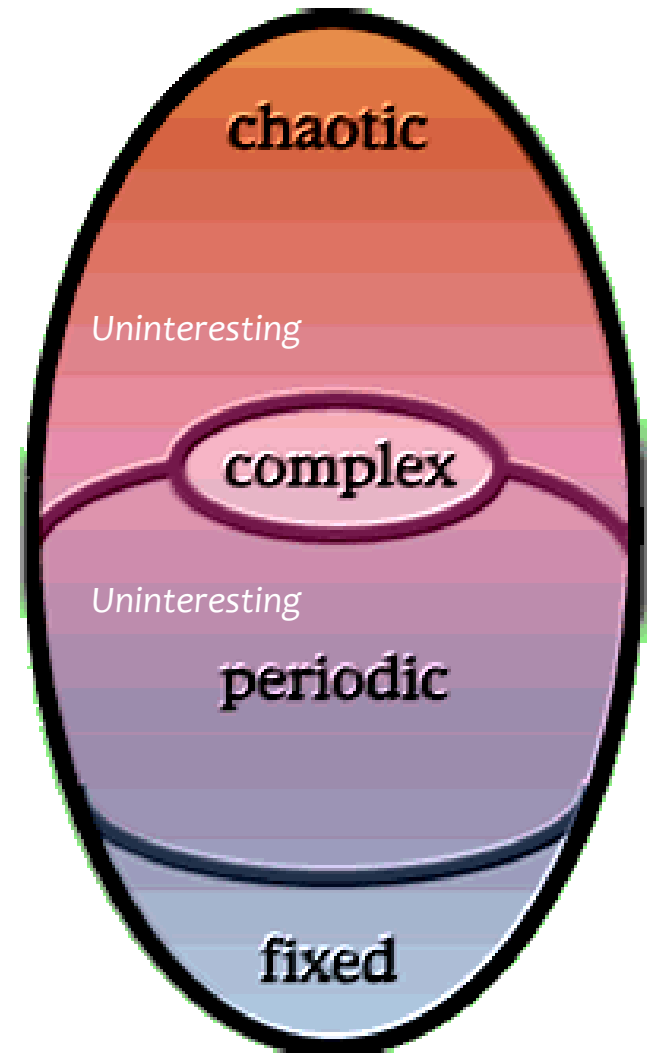
Static form can be characterized as $f(x) = 0$; balance means zero gradient $df(x)/dx = 0$. In dynamic balance underlying flows compensate each other; thermodynamic balance (or death) means that all flows have ceased.

- Even though the visible system may seem to consist of algebraic dependencies among variables, underneath there are (fast) dynamic processes
- These are typically (generalized) diffusion processes
- Otherwise, finding the balance cannot be explained in decentralized terms



Opposite intuitions #5

- The traditional intuition concerning complex systems says they are extremely unstable, always being at the “edge of chaos”
- “Steady state means death”
 - Erwin Schrödinger: Life is as far as possible from balance
 - Ilya Prigogine: The essence of life is in dissipative processes
- **Now:** The *static* and *dynamic* balances are different things!



- Non-balance thinking, emphasizing change, is very Western style
- Eastern wisdom takes harmony = balance as the underlying goal and ideal (philosophy, medicine, ...)
- The essence is in more or less fixed patterns, or *attractors* of dynamics

Chinese symbol for *air* or *vapor*; also meaning the *mystical ordering principle*



Cybernetic model – Intermediate summary

- The role of balances is crucial when constructing neocybernetic models; indeed, the emergent patterns that are to be modeled are “structures of stability” (see later)
- The **neocybernetic model** is a *model of balances*, or, if put in a more accurate way, it is a *balanced model of balances* (*higher-order balance*) taking into account also the nature of the environment (as determined by the statistical signal properties)
- The neocybernetic model is a *map of the relevant behaviors* corresponding to the observed environment, determining the *behavioral spectrum* of the system (where behavior means reactions to environmental excitations)



-
- In a nonlinear system, uniqueness of the balance cannot be assumed; indeed, the neocybernetic model covers the spectrum of alternatives or potential balances, as determined by the environment
 - The neocybernetic model is a **model over the local minima** rather than a model of the global optimum (assuming that an appropriate cost criterion is defined; *see later*)
 - Traditionally, the single global optimum is searched for in analysis and in design; this results in theoretical deadlocks (compare to NP problems: Finding a large number of suboptimal solutions is typically much simpler)
 - Also nature has no centralized master mind; it is facing the same optimization problems, seldom finding the strictly optimal solution: In this sense, the model over the local minima better captures the possible alternatives and *essence* (Remember Heraclitus: “You cannot step in the same river twice”)



Physically meaningful or mathematically possible?

- In theory, stability extremely rarely is encountered in arbitrary systems
 - The probability of a “completely random” n 'th order linear continuous system to be stable is $1/2^n$
- Indeed: The neocybernetic model is limited to a very narrow class of all possible dynamics – to those that are *relevant* in nature
- It is assumed that stability is caused by some internal mechanisms; in cybernetic system these causal structures are constituted by *negative feedback loops* (see later)
- The negative feedbacks are *control structures*; the different dynamic equilibria result from changing inputs (“reference signals”) – thus ...



Systems of *thermodynamic consistency* ...

- The neocybernetic model is a **model-based (adaptive) controller** trying to compensate the disturbances coming from the environment; further, this can be extended:
- The neocybernetic model is a **means of reaching maximum entropy** (or “**heat death**”) of the environment! (These things are all discussed *later*)
- In addition, there are also other views available that contradict traditional intuitions; indeed:
- The neocybernetic model is a **model of inverse thinking**: For example, the relationships are “pancausal” rather than unidirectional; it is *freedoms* rather than *constraints* that are modeled, etc.



... Intuitions applicable in different fields!

- Intuitions can efficiently be exploited: **Analogues** can be extended to partial differential equation models
- First, a *mechanical analogue*:
 - The neocybernetic model is an **elastic system**, where the **internal tensions compensate the external forces**. The deformations are proportional to the forces (behaving like a *steel plate*) whatever is their physical manifestation
- Then, an *electrical analogue*:
 - The neocybernetic model is a model where neighboring cybernetic systems can be managed: There is maximum power transfer among the systems when they are matched so that their **input and output impedances are equal**



“Pallas Athene Hypothesis”

Compare to “Gaia Hypothesis”

- It is by no means self-evident that mathematics can be used to explain nature; however, this far, it has been astonishingly efficient
- Here this optimistic belief is taken as the starting point: *The end of science is not yet there; complex systems can be modeled*

Is this justified?

- *If it is, there exist strong modeling guidelines to be followed ...*



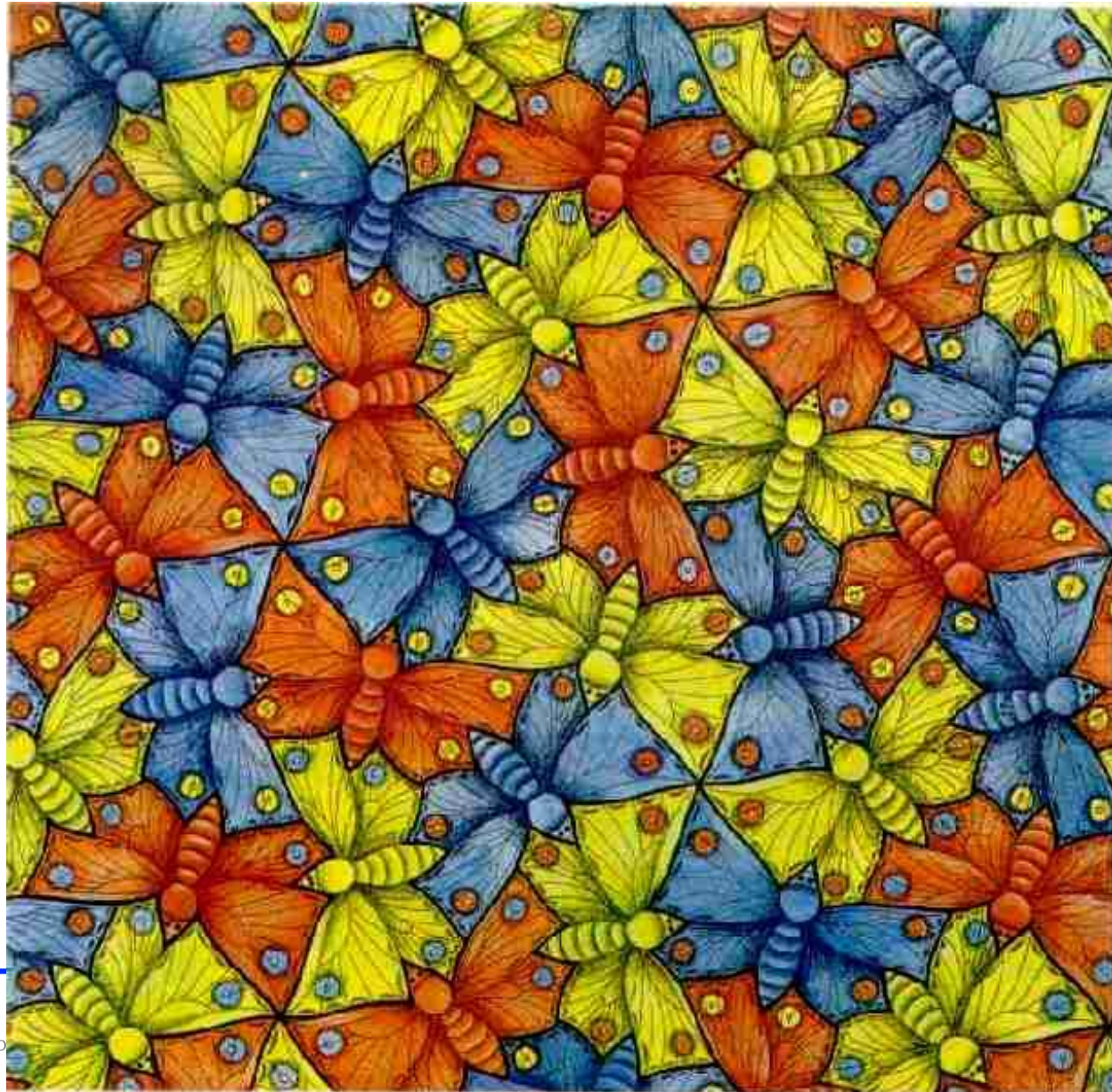
Additional intuition #1: *Determinism*

- The above stability studies did not yet give guidelines to select the model structure – how to get forward?
- Here – apply the “Pallas Athene Hypothesis”: Theory of complex systems **does** exist, so that systems have to be qualitatively more or less uniquely determined in each case (“free will” is a fallacy!)
- The parameters are to be optimized in some sense, so that the representation is unique within that model framework
- The neocybernetic model is (as it turns out) a “**mirror image**” of its environment, being itself a **model of the environment**, capturing relevant behavioral patterns as manifested in data (in a more or less unique manner).



Symmetry – and beauty?

- A system is surrounded by other systems
- Symmetricity: Environment and a system have to be interchangeable in the models



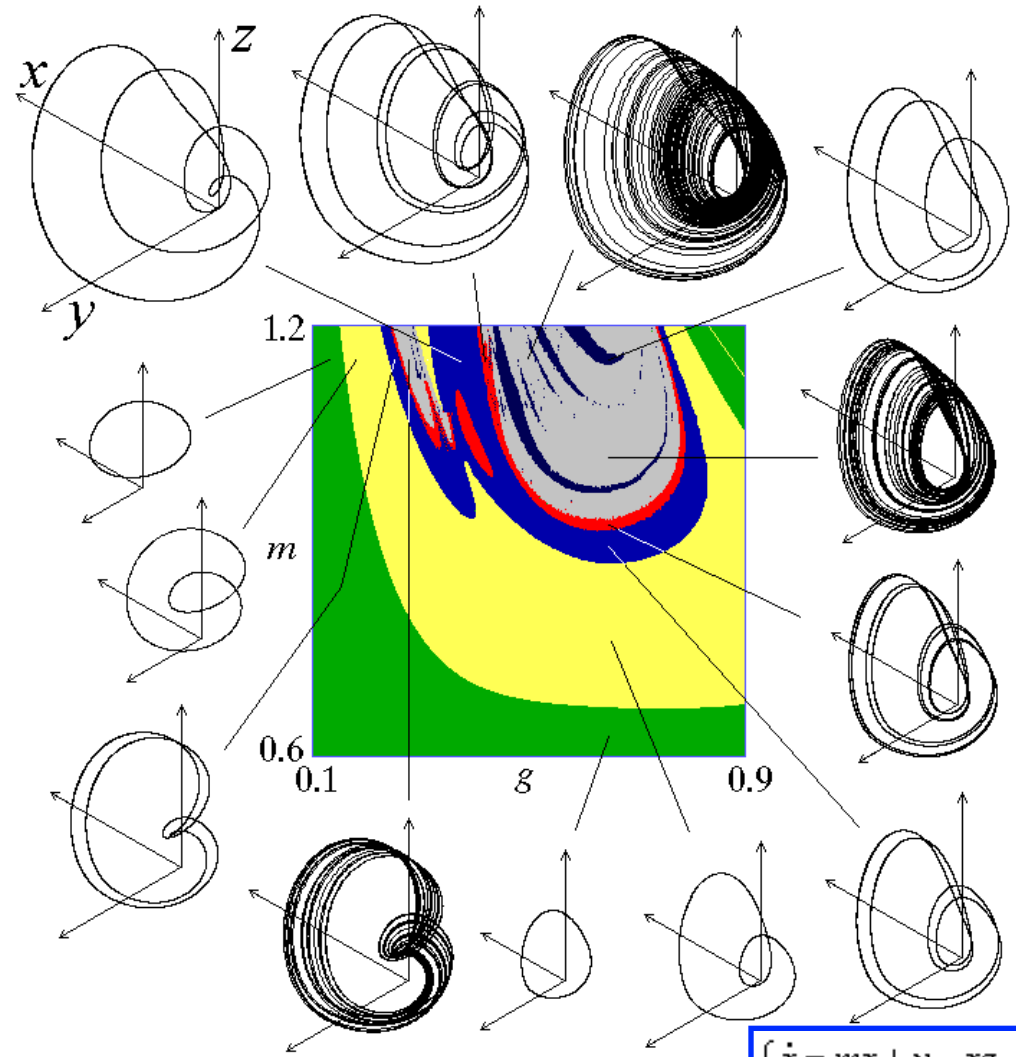
Additional intuition #2: *Linearity*

- System theoretic intuition: **Linearity** is essential in large-scale models – otherwise, no scalability and no predictability, etc.
– that is, no general theory of complex systems could exist!
- Here – again apply the “Pallas Athene Hypothesis”:
Theory of complex systems **does** exist, so that models to be created have to be *basically linear*
- Deficiencies in expressional power are compensated by high-dimensionality and interactions, resulting in stochastic, high-dimensional, dynamic feedback models
- Linearity can be motivated in cybernetic steady-state models when it is assumed that only minor deviations take place around the balance of a smooth nonlinear dynamics



Opposite intuitions #6

- Nonlinearity = always the basic starting point in all studies of chaos and complexity theory!
- Without nonlinearity qualitatively new phenomena cannot emerge – this IS true
$$"f(a+b) = f(a) + f(b)"$$
- **Now:** The *final state* can be studied without process nonlinearities



$$\begin{cases} \dot{x} = mx + y - xz \\ \dot{y} = -x \\ \dot{z} = -gz + 1(x)x^2 \end{cases}$$



Conclusion – to be utilized ...

- Intuition #1: Balance pursuit
 - Theoretical understanding: In steady state one can directly attack the emergent pattern and forget about the details of complex processes
 - Pragmatic understanding: Only in stable conditions, when fast phenomena have ceased, something fragile can emerge
- Intuition #2: Linearity pursuit
 - Theoretical understanding: Natural processes are smooth, and around the equilibrium (locally) linear
 - Pragmatic understanding: Linearity is the only way to scalability and out from toy worlds
- Motivations for both approaches
 - Heuristics: There is evolutionary advantage!?
 - Theory: Stronger tools are available for analysis and synthesis
 - Practice: It is clever to first study what can be reached with simple approaches – the assumptions can be relaxed (see later)



Opposite intuitions #7

- Philosophically speaking, traditional science is based on *Cartesian dualism*, or the distinction between the observing **subject** and the observed **object** (or *mind* and *matter*)
- Now, on the other hand, subjects and objects get mixed, as all entities are active – and because of pancausality, observation disturbs the observed
- However, there will be *another kind of dualism* introduced: it turns out that “emformation” (free information or energy) determines the structure wherein the matter is manifested
- Later on, when signals are analyzed in *frequency domain*, also Kantian *transcendental idealism* becomes challenged, as observations are not spatially and temporally determined.



Why linearity? – Bonus example

- System theoretic intuition also is: “To find scalable models for truly large systems, the model structure must be linear” (however, this constraint can be relaxed *later*)
- Why? Let us study an example – assume that

$$s(k+1) = f(As(k))$$

where

$$f_i(s) = \begin{cases} s_i, & \text{when } s_i > 0 \\ 0, & \text{when } s_i \leq 0. \end{cases}$$

- This is almost linear ... it even looks simpler than a linear model (only the first quadrant is employed)
- What kind of dynamics is possible here?



-
- Computability theory: Any algorithm can be constructed using only increment and decrement operations and conditional branches
 - Each of these commands can be implemented using the given dynamic model!
 - Matrix A represents the program structure, and s represents the program “snapshot” (program counter + variable values)
 - Initial state (variable values) given in $s(0)$
 - One program step takes one to two steps in the system
 - If the program halts, the system state converges; the resulting variable values can be read in s



- For example, the *parity function* can be coded as corresponding to

```

1  VAR X = x      input
2  VAR Y = 0      output
3  IF X > 0        entry point
   THEN X SUB 1 Y ADD 1 GOTO 6
   ELSE GOTO END
6  IF X > 0
   THEN X SUB 1 Y SUB 1 GOTO 3
   ELSE GOTO END

```

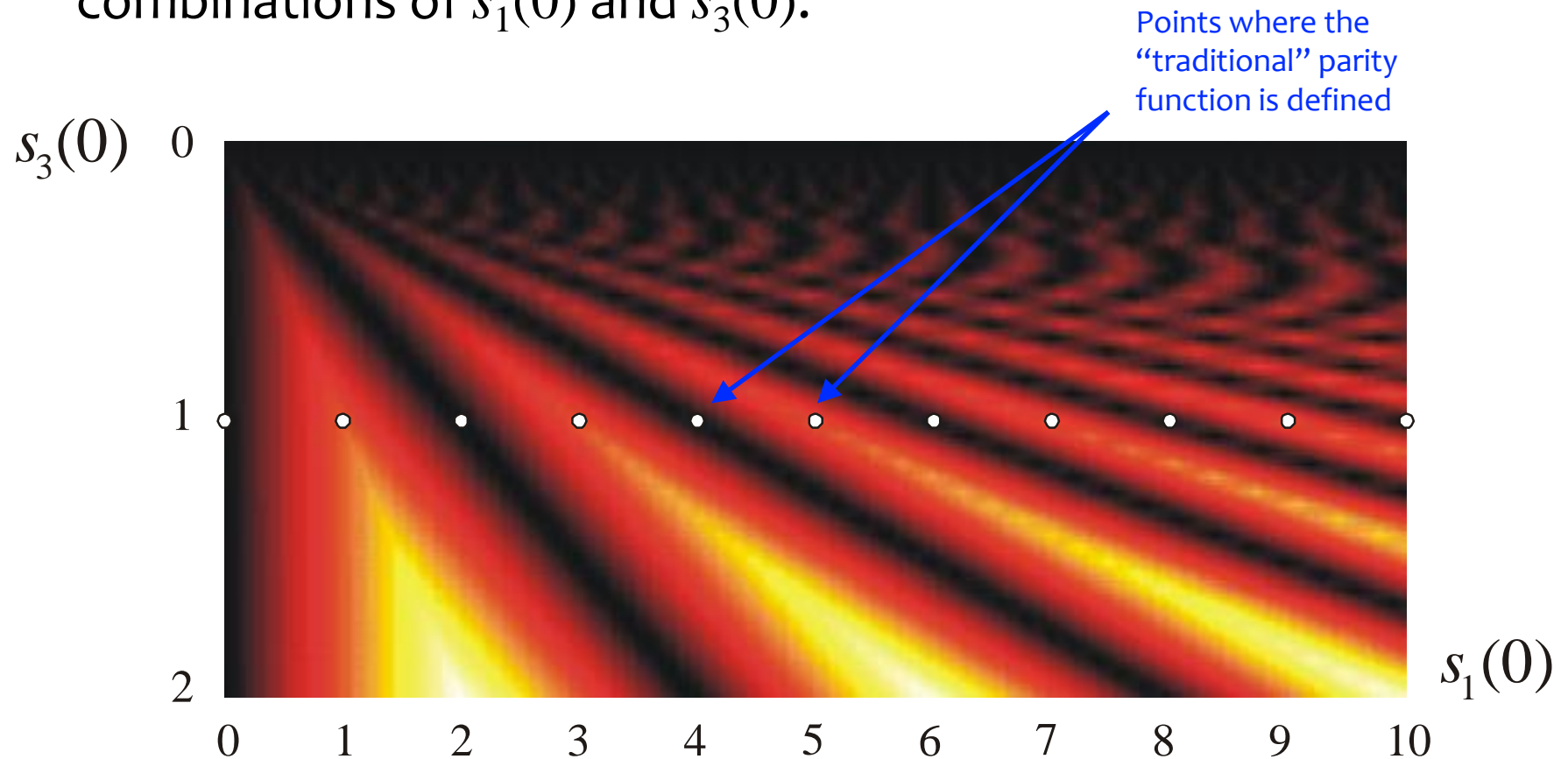
$$A_{\text{parity}} = \begin{pmatrix} + & & - & + & - & + \\ & + & + & - & - & + \\ & & & & + & - \\ & & + & & & \\ - & & + & & & \\ & & & + & - & \\ & & & & + & \\ & & & & & + \end{pmatrix}$$

$$s(0) = \begin{pmatrix} x \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$



“Generalized parity function”

- Eventual steady state values of s_2 corresponding to different combinations of $s_1(0)$ and $s_3(0)$:

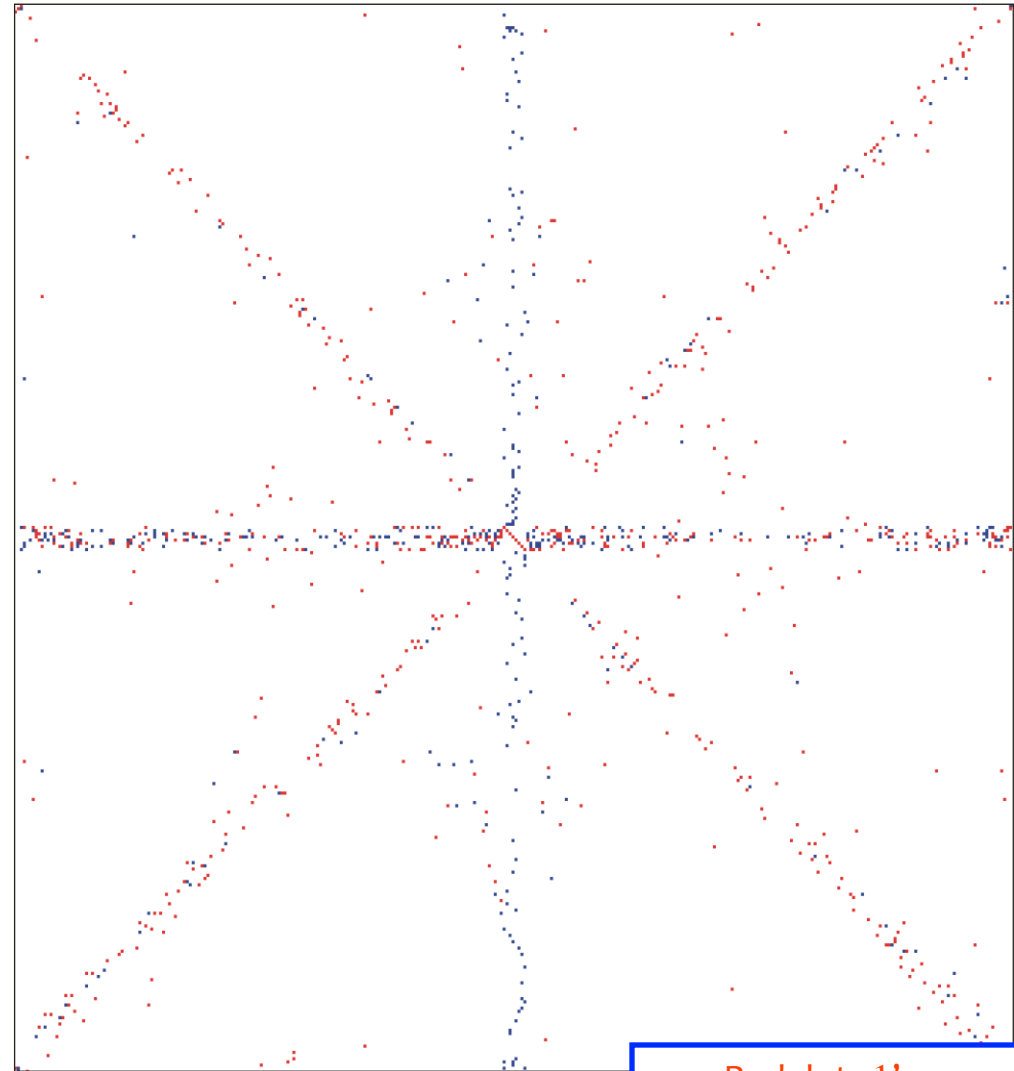


Universal machine implemented

- If the system is selected as shown here, there is no algorithm to say for all inputs $s(0)$ whether the system is stable ...

$A =$

- “If the algorithm claims that the iteration would remain bounded, it will not, and vice versa”!



Red dots 1's,
blue dots -1's,
other 0's



Universality and undecidability

- A nonlinear high-dimensional system can implement any imaginable algorithm
- Gödel's *incompleteness theorem* applies
- There will never exist a general theory of nonlinear systems!
- When searching for a general model structure for complex systems, only for (essentially) linear systems one has hope.



Kurt Gödel

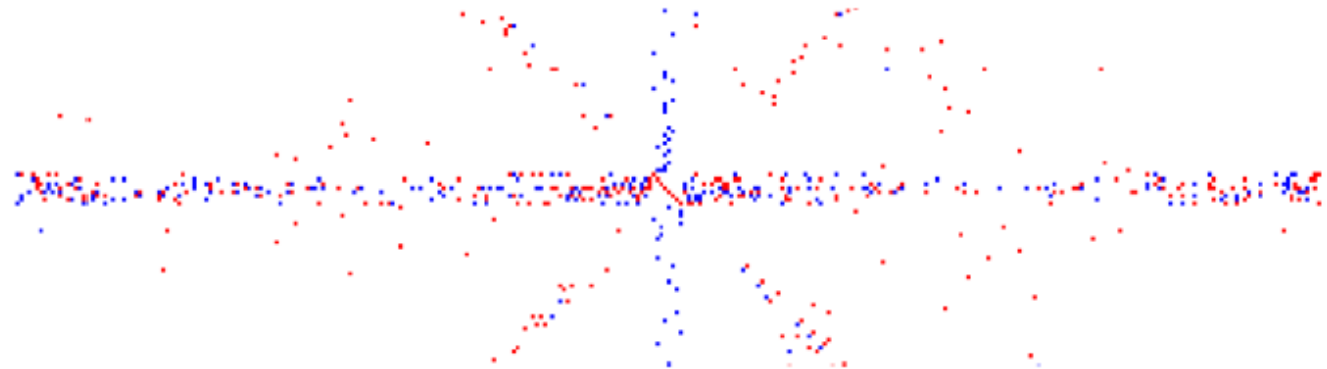


Report 133

- Illustrates the problems encountered in nonlinear systems

ON THE UNIVERSALITY AND UNDECIDABILITY IN DYNAMIC SYSTEMS

Heikki Hyötyniemi



NEW

- This example also illustrated that infinity can be addressed in finite space
- It showed, too, that structured representations can be implemented through iteration with minor *a priori* complexity
- So, there is motivation for cybernetic approaches: *Loops (proper feedback) can do wonders*
- To what extent is the expressional power caused by the *nonlinearity*, and to what extent by the *dynamic nature* alone?!



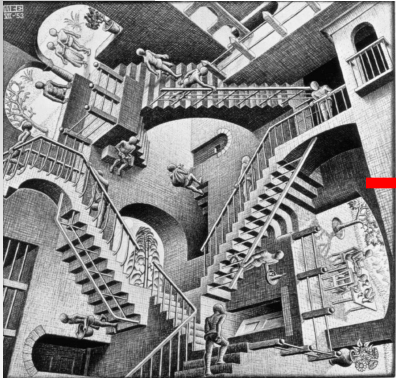
Modeling strategy – applied next time

- Linear models are traditionally regarded as deficient, inferior, models for pragmatic uses – where does that come from?
 - The typical engineering-like route: Construct the model as exactly as possible, applying nonlinearities where appropriate
 - After that, locally linearize the model to boost applications; unfortunately, in this process the connections to the real system are lost
- Now: Again, apply first physical principles exactly, never going to approximations
 - The application domain must not include explicit nonlinearities! – Yes, there exist such complex domains
 - Modeling strategy to be followed: Avoid introducing nonlinearities if it is not *absolutely necessary*



Instead, apply understanding of dynamic systems
and control + engineering intuitions about real
life constraints and non-idealities

- Stairs to step deeper are now available?



Iterate after the course!

